

The solutions to the Spring 2009 MFE/3F exam begin on the next page.

B.4 Solutions to Exam MFE/3F, Spring 2009

The questions for this exam may be downloaded from

<http://www.soa.org/files/pdf/edu-2009-05-mfe-exam.pdf>

1. [Section 4.2] The multipliers for up and down moves and the risk-neutral probability are

$$u = e^{(r-\delta+0.5\sigma)} = e^{0.05-0.05+0.3} = e^{0.3} = 1.34986$$

$$d = e^{-0.3} = 0.74081$$

$$p^* = \frac{1}{1+e^\sigma} = \frac{1}{1+e^{0.3}} = 0.425557$$

where we've used equation (3.6) to calculate p^* , since this tree is the forward tree. The resulting stock prices are shown in Figure B.3.

At the ending nodes, the option only pays off at the highest node. Pulling back one year:

$$C_u^{\text{tentative}} = e^{-0.05}(0.425557)(82.2119) = 33.2796$$

However, 33.2796 is less than the exercise value of 34.9859, so $C_u = 34.9859$. Then

$$C = e^{-0.05}(0.425557)(34.9859) = \boxed{14.1623} \quad (\text{E})$$

2. [Sections 13.1 and 13.2] The average of the 12 stock prices is

$$\frac{105 + 120 + \cdots + 110 + 115}{12} = 110$$

so the Asian call option pays 10.

The up-and-out pays nothing since the 125 barrier was hit; the up-and-in pays 5 since the barrier of 120 was hit and $115 - 110 = 5$. The answer is (B).

3. [Lesson 3] The risk-neutral probability of an up is

$$p^* = \frac{e^{r-\delta} - d}{u - d} = \frac{e^{0.05-0.10} - 0.8}{1.1 - 0.8} = 0.504098$$

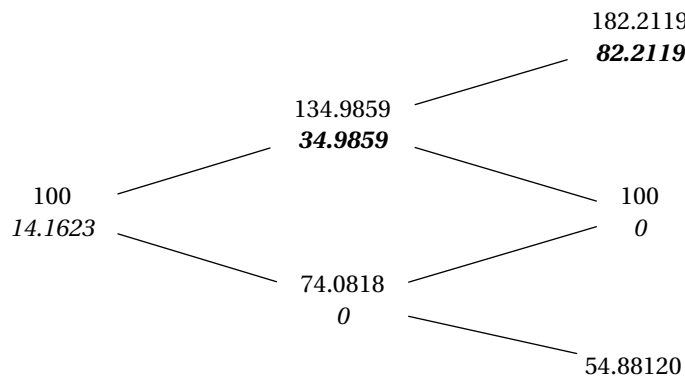


Figure B.3: Binomial tree for S09:1

so the value of the call is $e^{-0.05}(0.504098)(55 - 50) = 2.3976 > 1.90$, so buy the option and sell shares of WWW. When buying a call, shares must be sold since a call results in buying stock if it pays off. Lend the extra money. This is choice **(B)**.

The official solution also works this out with the replicating portfolio.

4. [Section 14.1] As listed in Table 14.1, the appropriate formula for this asset-or-nothing put option $S | S < K$ is $Se^{-\delta t}N(-d_1)$.

$$d_1 = \frac{\ln(1/0.6) + 0.025 - 0.02 + 0.5(0.2^2)}{0.2} = 2.68$$

$$N(-d_1) = 0.0037$$

$$S | S < K = 1000e^{-0.02}(0.0037) = 3.627$$

Multiplying by one million, the answer is **(D)**.

5. [Section 18.1] The zero-coupon bond is worth $e^{-0.18} = 0.83527$ at the top node, $e^{-0.12} = 0.88692$ at the two middle nodes, and $e^{-0.06} = 0.94176$ at the bottom node, so the option pays off at the top 3 nodes. Discounting with probabilities to the initial node:

$$P = (0.7^2)e^{-0.12-0.15}(0.9 - 0.83527) + (0.7)(0.3)e^{-0.12-0.15}(0.9 - 0.88692) + (0.7)(0.3)e^{-0.12-0.09}(0.9 - 0.88692)$$

$$= 0.02421 + 0.00210 + 0.00222 = \boxed{0.029} \quad \text{(E)}$$

6. [Section 17.1] The derivatives of $Y(t)$ are

$$Y_X = -\frac{1}{X^2} \qquad Y_{XX} = \frac{2}{X^3} \qquad Y_t = 0$$

By Itô's Lemma,

$$dY(t) = Y_X dX + 0.5Y_{XX}(dX)^2 + Y_t dt$$

$$= -\frac{8 - 2X(t)}{X(t)^2} dt - \frac{8}{X(t)^2} dZ(t) + \frac{64}{X(t)^3} dt$$

with the last term resulting from $(dX(t))^2 = (8 dZ(t))^2 = 64 dt$. Ignore the $dZ(t)$ term, since we don't need that for this question. The coefficient of dt is

$$-Y(t)^2 \left(8 - \frac{2}{Y(t)} \right) + 64Y(t)^3 = -8Y(t)^2 + 2Y(t) + 64Y(t)^3$$

so $\alpha(y) = -8y^2 + 2y + 64y^3$, and $\alpha(1/2) = -8(0.25) + 2(0.5) + 64(0.125) = \boxed{7}$. **(D)**

7. [Lesson 5] Tricky question: even though probabilities, assumptions, and payoffs don't change, the answer is not A because the risk-neutral probability changes.

The second method in the official solution is easier, and that is what I will use here.

From the information given, \$1.13 is $2p^*$ discounted one year, or

$$1.13 = 2 \left(\frac{e^r - d}{u - d} \right) e^{-r} = 2 \left(\frac{e^r - 0.8}{0.4} \right) e^{-r} = 5 - 2e^{-r}$$

$$e^{-r} = \frac{5 - 1.13}{2} = 1.935$$

In the revised tree,

$$C = 2 \left(\frac{e^r - 0.6}{0.6} \right) e^{-r} = \frac{10}{3} - 1.935 = \boxed{1.3983} \quad \text{(D)}$$

8. [Section 17.2] By the Black-Scholes Equation (17.4),

$$0.5S^2\sigma^2 V_{SS} + V_t + V_S S(r - \delta) = rV$$

For $V = e^{rt} \ln(S(t))$,

$$\begin{aligned} V_S &= \frac{e^{rt}}{S(t)} & V_{SS} &= -\frac{e^{rt}}{S(t)^2} & V_t &= rV \\ -0.5e^{rt}\sigma^2 + (r - \delta) + rV &= rV \\ -0.5\sigma^2 + r - \delta &= 0 \\ \delta &= r - 0.5\sigma^2 \end{aligned}$$

Plugging in the values we're given, $\delta = 0.055 - 0.5(0.3^2) = \boxed{0.01}$. (B)

The official solution has the following alternative. The risk-neutral expectation of the prepaid forward price of the security must equal the current value of the security. The current value ($t = 0$) of the security is $\ln(S(0))$, while the prepaid forward price of it is e^{-rt} times its price, or $\ln(S(t))$. So $\ln(S(0)) = \mathbf{E}[\ln(S(t))]$ for all t . But $\ln(S(t))$ has a normal distribution with mean $r - \delta - 0.5\sigma^2$ under the risk-neutral measure, so $r - \delta - 0.5\sigma^2 = 0$.

9. [Subsection 1.2.5] This question requires a currency translation and put-call parity.

If you do put-call parity first, then we calculate the value of a four-year dollar-denominated European call option on yen with strike price \$0.008:

$$\begin{aligned} 0.0005 - C &= 0.008e^{-4(0.03)} - 0.011e^{-4(0.015)} = -0.00326405 \\ C &= 0.0005 + 0.00326405 = 0.00376405 \end{aligned}$$

Then multiply by 125 since we need a put on 1 dollar or 125(0.008) dollars, and divide by 0.11 to translate the currency into dollars: $\yen0.00376405(125/0.11) = \boxed{\yen42.7733}$. (E)

If you want to do the currency translation first: a four-year put option to pay \$0.008 and get ¥1 is multiplied by 125 to make it pay \$1 and get ¥125, and then the price is \$125(0.0005). The price in yen is \$125(0.0005)/0.11 = ¥5.6818. By put-call parity, a put option on dollars to pay ¥125 and get \$1 is worth:

$$\begin{aligned} P - 5.6818 &= 125e^{-4(0.015)} - (1/0.11)e^{-4(0.03)} = 37.0914 \\ P &= 5.6818 + 37.0914 = \boxed{42.7733} \end{aligned}$$

10. [Lesson 17.3] As discussed in Example 17J, a risk-free portfolio is obtained by zeroing out the volatility, so if x_i is the amount to invest in asset i in this question, then

$$0.2x_1 - 0.25x_2 = 0$$

which implies $x_1 = (5/4)x_2$. Since $x_1 + x_2 = 1000$, it follows that $x_1 = (5/9)(1000) = \boxed{555.56}$. (C)

11. [Section 17.5] The expected value of $S(1)^a$ is calculated from formula (17.9):

$$\mathbf{E}[S(1)^a] = S(0)^a e^{a(a-\delta)+0.5a(a-1)\sigma^2}$$

and is equal to 1.4. We are being asked for the prepaid forward price of $S(1)^a$, or formula (17.11):

$$F_{0,1}^P(S(1)^a) = S(0)^a e^{-r+a(r-\delta)+0.5a(a-1)\sigma^2}$$

The quotient of the second formula over the first is

$$\frac{F_{0,1}^P(S^a)}{\mathbf{E}[S(1)^a]} = e^{-r-a(\alpha-r)}$$

so we need to back out a . The stochastic differential equation for $S(t)$ has $\alpha = 0.05$ and $\sigma = 0.2$.

$$\begin{aligned}\mathbf{E}[S(1)^a] &= S(0)^a e^{a(\alpha-\delta)+0.5a(a-1)\sigma^2} \\ \ln 1.4 &= a \ln 0.5 + 0.3 + (0.05a + 0.02a(a-1)) \\ 0.02a^2 + (\ln 0.5 + 0.03)a - \ln 1.4 &= 0 \\ a &= -0.49985\end{aligned}$$

The other solution to the quadratic is rejected since it is positive. Then, with $r = 0.03$,

$$F_{0,1}^P(S^a) = 1.4e^{-0.03+0.49985(0.05-0.03)} = \boxed{1.372} \quad (\text{C})$$

12. [Subsection 1.2.1 and Lesson 2]

I. $C(50, T)$ is worth more than $C(55, T)$ since calls decrease in value with increasing strike prices, and the difference in values is less than $5e^{-rT}$ because if you buy $C(50, T)$ and sell $C(55, T)$, the most you can get is 5 at time T , which is worth $5e^{-rT}$ at time 0. ✓

II. By put-call parity,

$$P(50, T) - C(50, T) = 50e^{-rT} - S$$

and $P(45, T) > P(50, T)$, so $P(45, T) - C(50, T) \leq 50e^{-rT}$, and is certainly also less than $55e^{-rT}$, but the left side inequality is incorrect. ✗

III. We proved the right hand inequality in (II). The left hand inequality follows from

$$P(45, T) - C(45, T) = 45e^{-rT} - S$$

and $C(45, T) \geq C(50, T)$. ✓ (E)

The answer choices are asymmetric, something which was not allowed on previous exams, but II and III cannot both be true, so they perhaps were forced to make them asymmetric.

13. [Section 9.1] By the Black-Scholes formula, the value of the option at time 8 months, with 4 months left to expiry, is

$$\begin{aligned}d_1 &= \frac{\ln(85/75) + (0.05 + 0.5(0.26^2))(1/3)}{0.26\sqrt{1/3}} = 1.0199 \\ d_2 &= 1.0199 - 0.26\sqrt{1/3} = 0.8698 \\ N(d_1) &= N(1.02) = 0.8461 \\ N(d_2) &= N(0.87) = 0.8078 \\ C &= 85(0.8461) - 75e^{-0.05(1/3)}(0.8078) = 12.33\end{aligned}$$

The eight-month holding profit, taking into account the eight-month interest cost on the original investment of 8, is $12.33 - 8e^{0.05(2/3)} = \boxed{4.06}$. (A)

14. [Section 18.2.2] First, in a Black-Derman-Toy tree, the vertical ratios between interest rates are constant, so $0.8/r_{ud} = r_{ud}/0.2$ and therefore $r_{ud} = 0.4$.

By formula (18.1), the forward's price is $P(0,3)/P(0,2)$. We calculate these in terms of the initial interest rate r_0 . In the following, $P_u(1,3)$ is the price of a two-year bond at the upper node after one year, and $P_d(1,3)$ is the price of a two-year bond at the lower node after one year.

$$\begin{aligned} P(0,2) &= \frac{0.5}{1+r_0} \left(\frac{1}{1.6} + \frac{1}{1.3} \right) = \frac{0.697115}{1+r_0} \\ P_u(1,3) &= \frac{0.5}{1.6} \left(\frac{1}{1.8} + \frac{1}{1.4} \right) = 0.396825 \\ P_d(1,3) &= \frac{0.5}{1.3} \left(\frac{1}{1.4} + \frac{1}{1.2} \right) = 0.595238 \\ P(0,3) &= \frac{0.5}{1+r_0} (0.396825 + 0.595238) = \frac{0.496032}{1+r_0} \end{aligned}$$

The answer to the question is $1000F_{0,2}(P(2,3)) = 1000(0.496032/0.697115) = \boxed{711.55}$. (E)

The question omitted r and used high interest rates to try to trick you. Some students mistakenly thought that the forward price is the probability-weighted average of the one-year bond prices at the end of two years, or

$$0.25 \left(\frac{1}{1.8} \right) + 0.5 \left(\frac{1}{1.4} \right) + 0.25 \left(\frac{1}{1.2} \right) = 0.70437$$

and 704.37 is not one of the five answer choices.

15. [Section 20.3] We recognize the model as a Vasicek model with $a = 0.1$ and $\sigma = 0.05$. The Sharpe ratio is deduced by comparing the risk-neutral process to the true process; the Sharpe ratio ϕ times σdt is added to go from the latter to the former, and the risk-neutral process is $0.005 dt$ more than the true process, so $\phi = 0.005/0.05 = 0.1$. Then

$$\begin{aligned} B(t, T) &= \frac{1 - e^{-a(T-t)}}{a} \\ B(2, 5) &= \frac{1 - e^{-0.1(3)}}{0.1} = 2.59182 \\ q(0.04, 2, 5) &= B(2, 5)\sigma = 2.59182(0.05) \\ \frac{\alpha(0.04, 2, 5) - 0.04}{q(0.04, 2, 5)} &= 0.1 \\ \alpha(0.04, 2, 5) &= 2.59182(0.05)(0.1) + 0.04 = \boxed{0.05296} \quad (\text{C}) \end{aligned}$$

16. [Section 16.1.2] The parameters m and ν of the lognormal distribution of the stock price after 9 months are

$$\begin{aligned} m &= (\alpha - \delta - 0.5\sigma^2)(t) = (0.1 - 0.5(0.3^2))(0.75) = 0.04125 \\ \nu &= \sigma\sqrt{t} = 0.3\sqrt{0.75} = 0.2598 \end{aligned}$$

So the probability that the stock price is more than 125 is

$$1 - N\left(\frac{\ln(125/100) - 0.04125}{0.2598}\right) = 1 - N(0.70) = \boxed{0.242} \quad (\text{A})$$

17. [Subsection 10.1.1] The formula for a put's delta is $e^{-\delta t} (N(d_1) - 1)$ (equation (10.3)), and since $N(-d_1) = 1 - N(d_1)$, this is the same as $-e^{-\delta t} N(-d_1)$. Since $\delta = 0$,

$$\begin{aligned} -N(-d_1) &= -0.4364 \\ d_1 &= 0.16 \end{aligned}$$

We set up the quadratic equation for σ .

$$\begin{aligned} \frac{r + 0.5\sigma^2}{\sigma} &= 0.16 \\ 0.012 + 0.5\sigma^2 &= 0.16\sigma \\ 0.5\sigma^2 - 0.16\sigma + 0.012 &= 0 \end{aligned}$$

The two solutions are $\sigma = 0.12, 0.20$. Higher volatility leads to higher put prices, so if 20% satisfies (i), 12% certainly does, and since the answer is unique, we know the answer has to be **12%**. (A) The official solution shows how you can verify $\sigma < 0.14$ without explicitly valuing the put with Black-Scholes formula for $\sigma = 0.2$; you can express the Black-Scholes formula for the put over the stock price in terms of $N(d_1)$ and $N(d_2)$, set $d_2 = d_1 - \sigma$, and then get an upper bound for σ .

18. [Section 17.3] The Sharpe ratios of the two stocks must equal. Let α_i and σ_i be the rates of return and volatilities of the stocks with prices S_i . Then

$$\begin{aligned} \alpha_1 &= 0.1 + 0.5(0.2^2) = 0.12 \\ \alpha_2 &= 0.125 + 0.5(0.3^2) = 0.17 \\ \frac{0.12 - r}{0.2} &= \frac{0.17 - r}{0.3} \\ 0.036 - 0.3r &= 0.034 - 0.2r \\ 0.1r &= 0.002 \\ r &= \mathbf{0.02} \quad (\text{A}) \end{aligned}$$

19. [Section 9.1] The notation $\text{Var}(\ln F_{t,1}^P(S))$ looks a little weird, but it denotes the variance of the prepaid forward from time t to time 1, as of time 0, as a function of t . When $t = 0$, there is no volatility since the prepaid forward price is known, but as time goes on, the volatility, from the perspective of valuation at time 0, keeps growing. So $\sigma^2 = 0.01$ and $\sigma = 0.1$. We can now use the Black-Scholes formula on the prepaid forward of the stock, whose value at time 0 is

$$50 - 5e^{-rt} = 50 - 5e^{-0.75(0.12)} = 45.43034$$

Plugging into the prepaid forward version of the Black-Scholes formula, equation (9.1),

$$\begin{aligned} d_1 &= \frac{\ln(45.43034/45e^{-0.12}) + 0.5(0.1^2)}{0.1} = 1.3452 \\ d_2 &= 1.3452 - 0.1 = 1.2452 \\ N(-d_1) &= N(-1.35) = 0.0885 \\ N(-d_2) &= N(-1.25) = 0.1056 \\ P &= 45e^{-0.12}(0.1056) - 45.43034(0.0885) = 0.1941 \end{aligned}$$

and 100 units have value **19.41**. (D)

20. [Section 12.2] The delta-gamma approximation is equation (12.2) without $h\theta$. Thus we have

$$\begin{aligned}\Delta\epsilon + 0.5\Gamma\epsilon^2 &= 2.21 - 2.34 = -0.13 \\ -0.181\epsilon + 0.5(0.035)\epsilon^2 &= -0.13 \\ 0.0175\epsilon^2 - 0.181\epsilon + 0.13 &= 0 \\ \epsilon &= \frac{0.181 \pm \sqrt{0.181^2 - 4(0.0175)(0.13)}}{0.035} = 9.5663, 0.7765\end{aligned}$$

The original stock price is $86 - \epsilon$. Using $\epsilon = 0.7765$ gets an original stock price of $S(0) = \boxed{85.2235}$. (C). Using 9.5663 gets a stock price below 80, violating (i). The delta-gamma approximation is invalid for such a large change, since the squared epsilon in the gamma term overwhelms delta in such a case.

$$6 + 1.5x = 4 + 2x$$

$$0.5x = 2$$

$$x = \boxed{4} \quad (\mathbf{E})$$

Student reports indicated that this question appeared on the Fall 2008 exam.

49. [Lesson 3] The binomial tree will have

$$u = e^{0.04(0.25) + 0.3\sqrt{0.25}} = e^{0.16}$$

$$d = e^{0.01 - 0.15} = e^{-0.14}$$

$$1 - p^* = \frac{1}{1 + e^{-\sigma\sqrt{h}}} = \frac{1}{1 + e^{-0.15}} \quad (\text{Equation (3.7)})$$

Note that $100u = 100e^{0.16} = 117.35 < 118$. When the option pays off at both nodes, it is optimal to exercise early since the option no longer has any risk. Let's assume the $K < 117.35$ so that the option does not pay at the upper node. For optimal early exercise, we need the current payoff to be worth more than the discounted payoff at the lower node, or

$$\begin{aligned} K - 100 &\geq e^{-0.01} (1 - p^*) (K - 100e^{-0.14}) = e^{-0.01} \left(\frac{K - 100e^{-0.14}}{1 + e^{-0.15}} \right) \\ &\geq \frac{Ke^{-0.01} - 100e^{-0.15}}{1 + e^{-0.15}} \\ (K - 100)(1 + e^{-0.15}) &\geq Ke^{-0.01} - 100e^{-0.15} \\ K(1 + e^{-0.15} - e^{-0.01}) &\geq 100(1 + e^{-0.15} - e^{-0.15}) = 100 \\ K &\geq \frac{100}{1 + e^{-0.15} - e^{-0.01}} = \frac{100}{0.870658} = 114.86 \end{aligned}$$

so the answer is $\boxed{115}$. (B)

50. [Section 7.2] The parameters of the lognormal ratio $S_{1/2}/S_0$ are

$$m = 0.5(0.15 - 0.5(0.35^2)) = 0.044375$$

$$v = 0.35\sqrt{0.5} = 0.247487$$

The upper bound of the 90% confidence interval for the normal distribution is $0.044375 + 1.645(0.247487) = 0.451491$. The upper bound of the 90% confidence interval for the stock price after six months is $0.25e^{0.451491} =$

$\boxed{0.393}$. (A)

51. [Section 8.1] A statistical calculator may help for this question.

We calculate $\ln(S_t/S_{t-1})$ for times $t = 2$ through $t = 7$.

Month	2	3	4	5	6	7
S_t/S_{t-1}	0.03637	-0.15415	0.13613	0.08701	-0.03390	0.06669

$\ln(S_t/S_{t-1})$ is assumed to follow a normal distribution. We estimate the mean of the normal as the sample mean $\mu = \bar{x} = 0.02303$ and the standard deviation as the sample standard deviation (dividing by $n - 1 = 5$) $\sigma = 0.10354$. These are per month. The monthly return is therefore $\alpha/12 = \mu + 0.5\sigma^2 = 0.02303 + 0.5(0.10354^2) = 0.02839$. The annual return is $\alpha = 12(0.02839) = \boxed{0.3406}$. (E) You can also reverse the order of these operations: first annualize μ (multiply by 12) and σ (multiply by $\sqrt{12}$) and then calculate $\alpha = \mu + 0.5\sigma^2$. It is strange that the answer is so far out of the ranges; they were thinking you'd forget to add $0.5\sigma^2$.

52. [Section 15.3] For 2 years, the mean and variance of the lognormal are

$$m = 2(0.15 - 0.5(0.3^2)) = 0.21$$

$$v = 0.30\sqrt{2} = 0.4243$$

Using the inversion method, the standard normal random numbers are $N^{-1}(0.9830) = 2.12$, $N^{-1}(0.0384) = -1.77$, and $N^{-1}(0.7794) = 0.77$. The resulting ratios of S_1/S_0 are

n_i	$m + n_i v$	$e^{m+n_i v}$
2.12	1.1094	3.0327
-1.77	-0.5410	0.5822
0.77	0.5367	1.7103

The average, multiplied by $S_0 = 50$, is $50(3.0327 + 0.5822 + 1.7103)/3 = \boxed{88.75}$. (C)

53. [Section 14.1] We can express the gap option as a standard European call with strike price 40 minus a cash-or-nothing option paying 5 if the stock price is above 40. Now the two properties of gamma to use are:

- Gamma is a linear function of the options, since it is a second derivative with respect to the stock price, so a linear combination of the two options will have a gamma that is the linear combination of the gammas.
- Gamma for a put equals gamma for the corresponding call, the one with the same strike price and expiry.

Thus gamma for a call plus gamma for a cash-or-nothing of 5 equals gamma for the gap call.

$$0.08 + \text{gamma cash-or-nothing} = 0.07$$

$$\text{gamma cash-or-nothing} = -0.01$$

For cash-or-nothing of 1000, gamma is $(1000/5)(-0.01) = \boxed{-2}$. (B)

Alternatively, write down all the information using our all-or-nothing notation:

$$\text{(ii) \& (iv): } 40 | S < 40 - S | S < 40 \quad \Gamma = 0.07$$

$$\text{(iii) \& (v): } S | S > 40 - 45 | S > 40 \quad \Gamma = 0.08 \quad (*)$$

and we need gamma for $1000 | S > 40$. Since we need the condition $S > 40$, we will replace the first equation above with a call equation:

$$S | S > 40 - 40 | S > 40 \quad \Gamma = 0.07 \quad (**)$$

Subtracting (*) from (**), we get $5 | S > 40$ has $\Gamma = -0.01$, so $1000 | S > 40$ has $\Gamma = 200(-0.01) = \boxed{-2}$. (B)

54. [Sections 1.2, 13.3, and 14.3] We want the value of an option with payoff $\max(17 - \min(2S_1(1), S_2(1)), 0)$, since the option holder will sell the cheapest stock for 17.

Let M be the value at time 1 of $\min(2S_1(1), S_2(1))$. Then the option given in (vi)—call its value V_1 —pays $\max(M - 17, 0)$ and the option we want to value—call its value V_2 —pays $\max(17 - M, 0)$. Since one option pays exactly when the other one doesn't, the difference, or $V_1 - V_2$, is equal to the present value of $M - 17$. The present value of 17 is $17e^{-0.05}$. The present value of M is an option paying the minimum of $2S_1$ and S_2 , which is $S_2 - C(S_2, 2S_1)$, where $C(S_2, 2S_1)$ is an exchange option which allows one to buy S_2 in exchange for $2S_1$. Let's value this exchange option. The relative volatility of the two stocks is the square root of

$$\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 = 0.18^2 + 0.25^2 - 2(-0.40)(0.18)(0.25) = 0.1309$$

and $\sqrt{0.1309} = 0.3618$. By Black-Scholes, since both dividend rates are 0,

$$d_1 = \frac{\ln(20/20) + 0.5\sigma^2}{\sigma} = 0.5(0.3618) = 0.1809$$

$$d_2 = 0.1809 - 0.3618 = -0.1809$$

$$N(d_1) = N(0.18) = 0.5714$$

$$N(d_2) = N(-0.18) = 0.4286$$

$$C(S_2, 2S_1) = 20(0.5714 - 0.4286) = 2.856$$

so the present value of M is $20 - 2.856 = 17.144$. Then

$$V_1 - V_2 = 17.144 - 17e^{-0.05}$$

$$1.632 - V_2 = 17.144 - 17e^{-0.05} = 0.9731$$

$$V_2 = 1.632 - 0.9731 = \boxed{0.659} \quad (\text{A})$$

55. [Section 9.3] We must back out σ . For an at-the-money future, $d_1 = -d_2$. Therefore,

$$\begin{aligned} 1.625 &= 20e^{-0.75(0.1)}(N(-d_2) - N(-d_1)) \\ &= 20e^{-0.075}(N(d_1) - (1 - N(d_1))) = 20e^{-0.075}(2N(d_1) - 1) \end{aligned}$$

$$2N(d_1) = \frac{1.625e^{0.075}}{20} + 1 = 1.087578$$

$$N(d_1) = 0.5438$$

$$d_1 = 0.11 = 0.5\sigma\sqrt{3/4}$$

$$\sigma = \frac{0.22}{\sqrt{3/4}} = 0.2540$$

Now we can value the option three months later using the Black formula. With six months to expiry:

$$d_1 = \frac{\ln(17.7/20) + 0.5(0.2540^2)(0.5)}{0.2540\sqrt{0.5}} = -0.5903$$

$$d_2 = -0.5903 - 0.2540\sqrt{0.5} = -0.7699$$

$$N(-d_1) = N(0.59) = 0.7224$$

$$N(-d_2) = N(0.77) = 0.7794$$

$$P(F, 20, 0.5) = 20e^{-0.5(0.1)}(0.7794) - 17.7e^{-0.5(0.1)}(0.7224) = \boxed{2.6649} \quad (\text{D})$$

56. [Sections 7.1 and 13.1] Nothing about this question requires knowledge of average strike options. You just have to evaluate the variance of $A(2)$.

The stock price is 5 times a lognormal random variable with parameters $m = (0.05 - 0.5(0.2^2))t = 0.03t$ and $v = 0.2\sqrt{t}$. Therefore,

$$A(2) = \frac{1}{2}(S(1) + S(2)) = \frac{S(0)}{2} \left(\frac{S(1)}{S(0)} + \frac{S(2)}{S(0)} \right)$$

While $S(1)/S(0)$ and $S(2)/S(0)$ are not independent because the periods $(0, 1)$ and $(0, 2)$ overlap, the variables $X = S(1)/S(0)$ and $Y = S(2)/S(1)$ are independent, so we set $S(2)/S(0) = XY$, and

$$A(2) = \frac{5}{2}(X + XY)$$

where X and Y are independent lognormal random variables with parameters $m = 0.03$ and $v = 0.2$.

We will use the two alternative methods as the official solution goes through: (1) calculating first and second moments, and (2) calculating variance of a sum directly.

First let's calculate the variance as the second moment minus the first moment squared. We can hold off the $5/2$ until the end.

$$\begin{aligned}\mathbf{E}[X + XY] &= \mathbf{E}[X] + \mathbf{E}[XY] = e^{0.05} + e^{0.10} \\ \mathbf{E}[(X + XY)^2] &= \mathbf{E}[X^2] + \mathbf{E}[(XY)^2] + 2\mathbf{E}[X^2Y]\end{aligned}$$

X and Y are independent, so the expectations may be factored. Also, $\mathbf{E}[Z^2] = e^{2m+2v^2}$ for a lognormal Z . So

$$\begin{aligned}\mathbf{E}[X^2] &= e^{2(0.03)+2(0.2^2)} = e^{0.14} \\ \mathbf{E}[(XY)^2] &= \mathbf{E}[X^2]\mathbf{E}[Y^2] = e^{0.14}e^{0.14} = e^{0.28} \\ \mathbf{E}[X^2Y] &= \mathbf{E}[X^2]\mathbf{E}[Y] = e^{0.14}e^{0.05} = e^{0.19}\end{aligned}\quad (*)$$

Therefore the variance is

$$\begin{aligned}\text{Var}(X + XY) &= e^{0.14} + e^{0.28} + 2e^{0.19} - (e^{0.05} + e^{0.10})^2 \\ &= 4.891903 - 2.156442^2 = 0.241661\end{aligned}$$

Multiplying by $(5/2)^2$, the answer is $(25/4)(0.241661) = \boxed{1.51}$. (A)

The other method is to calculate variance directly using the variance of a sum formula:

$$\text{Var}(X + XY) = \text{Var}(X) + \text{Var}(XY) + 2\text{Cov}(X, XY)$$

Since X and Y are independent, their product is a lognormal with $m = 0.06$ and $v^2 = 2(0.2^2) = 0.08$, so

$$\begin{aligned}\text{Var}(X) &= \mathbf{E}[X^2] - \mathbf{E}[X]^2 = e^{0.14} - e^{0.1} \\ \text{Var}(XY) &= e^{0.28} - e^{0.2}\end{aligned}$$

For the covariance, evaluate it as

$$\begin{aligned}\text{Cov}(X, XY) &= \mathbf{E}[X^2Y] - \mathbf{E}[X]\mathbf{E}[XY] \\ \mathbf{E}[X^2Y] &= e^{0.19} \quad \text{This was evaluated above, see (*).} \\ \mathbf{E}[X] &= e^{0.05} \\ \mathbf{E}[XY] &= e^{0.1} \\ \text{Cov}(X, XY) &= e^{0.19} - e^{0.15}\end{aligned}$$

So the variance of $X + XY$ is

$$\text{Var}(X + XY) = e^{0.14} - e^{0.1} + e^{0.28} - e^{0.2} + 2(e^{0.19} - e^{0.15}) = 0.241661$$

and multiplying by $(5/2)^2$, the answer again is $(25/4)(0.241661) = \boxed{1.51}$.

57. [Section 15.5] U_1 and U_5 get mapped to the first stratum or $[0, 0.25)$ and U_4 and U_8 get mapped to the last stratum, or $[0.75, 1)$, so the lowest number must come from U_1 or U_5 and the highest one from U_4 or U_8 . Of U_1 and U_5 , U_5 is lower. Using inversion, $N^{-1}(0.3172/4) = N(0.0793) = -1.41$. Of U_4 and U_8 , U_4 is higher. Using inversion, $N^{-1}(0.75 + 0.4482/4) = N^{-1}(0.8621) = 1.09$. The difference is $1.09 - (-1.41) = \boxed{2.50}$. (E)

58. [Section 15.4] $C(40)$ is not random, so the variance of the expression for $C^*(42)$ is

$$\hat{\text{Var}}(\hat{C}(42)) + \beta^2 \text{Var}(\hat{C}(40)) + 2\beta \text{Cov}(\hat{C}(40), \hat{C}(42))$$

The minimum of a quadratic is $\beta = -b/2a$, or $\text{Cov}(\hat{C}(40), \hat{C}(42)) / \text{Var}(\hat{C}(40))$. This is estimated with the sample covariance and variance. The simulated payoffs are

Price	Strike 40	Strike 42	Strike 40 squared	Product
33.29	0	0	0	0
37.30	0	0	0	0
40.35	0.35	0	0.1225	0
43.65	3.65	1.65	13.3225	6.0225
48.90	8.90	6.90	79.21	61.41
Sum	12.90	8.55	92.6550	67.4325

We don't have to discount by $e^{-0.25(0.08)}$ because the discount factor appears in both the numerator and denominator and cancels. Using standard formulas for covariance and variance, each multiplied by 5,

$$\beta = \frac{\text{Cov}(\hat{C}(40), \hat{C}(42))}{\text{Var}(\hat{C}(40))} = \frac{67.4325 - (12.90)(8.55)/5}{92.6550 - 12.90^2/5} = \boxed{0.764211} \quad (\text{B})$$

The official answer points out that you can perform the regression on a statistical calculator without going through the calculations here.

59. [Section 15.4] For each $\hat{C}(K)$, we discount the average simulated value.

$$\hat{C}(42) = e^{-0.08(0.25)}(8.55/5) = 1.6761$$

$$\hat{C}(40) = e^{-0.08(0.25)}(12.90/5) = 2.5289$$

$$C^*(42) = 1.6761 + 0.764211(2.7847 - 2.5289) = \boxed{1.8716} \quad (\text{B})$$

60. [Section 20.2] You're given that the limit of the price of a bond at infinity is $e^{-0.1T}$, so the yield of an infinitely lived bond is 0.1. The formula for this yield (see the last line of Table 20.3) is

$$\bar{r} = \frac{2ab}{a - \bar{\phi} + \gamma} = \frac{2ab}{(a - \bar{\phi}) + \sqrt{(a - \bar{\phi})^2 + 2\bar{\sigma}^2}}$$

and in our case, since

$$dr(t) = 0.1(0.11 - r(t))dt + 0.08\sqrt{r(t)}dZ(t)$$

we have $a = 0.1$, $b = 0.11$, $\bar{\sigma} = 0.08$. It is easier to let the unknown be $x = a - \bar{\phi}$ rather than $\bar{\phi}$, so that we don't have to expand $(a - \bar{\phi})^2$. Then

$$0.1 = \frac{2(0.1)(0.11)}{x + \sqrt{x^2 + 2(0.08^2)}} = \frac{0.022}{x + \sqrt{x^2 + 0.0128}}$$

$$1 = \frac{0.22}{x + \sqrt{x^2 + 0.0128}}$$

$$x + \sqrt{x^2 + 0.0128} = 0.22$$

$$x^2 + 0.0128 = (0.22 - x)^2 = x^2 - 0.44x + 0.0484$$

$$0.44x = 0.0484 - 0.0128 = 0.0356$$

$$x = 0.080909$$

Then $\bar{\phi} = 0.1 - x = 0.019091$. Since $\phi(r, t) = \bar{\phi} \sqrt{r}/\bar{\sigma}$, $c = \bar{\phi}/\bar{\sigma} = 0.019091/0.08 = \boxed{0.2386}$. (E) The official solution points out that you don't have to solve for $\bar{\phi}$; you can just plug in the five answer choices (after transforming them to $\bar{\phi}$ by multiplying by 0.08).

61. [Section 17.4] The relationship between the risk-neutral process and the true process is $\tilde{Z}(t) = Z(t) + \phi t$, where ϕ is the Sharpe ratio, as discussed in Section 17.4. Then

$$\mathbf{E}^*[Z(t)] = \mathbf{E}^*[\tilde{Z}(t)] - \phi t$$

where \mathbf{E}^* indicates expectation under the risk-neutral measure. Under the risk-neutral measure, $\tilde{Z}(t)$ is a Brownian motion, so $\mathbf{E}^*[\tilde{Z}(t)] = 0$. We are given $\mathbf{E}^*[Z(0.5)] = -0.03$, so $-0.5\phi = -0.03$ and $\phi = 0.06$. Now, $\phi = (\alpha - r)/\sigma$, and we see from the stock-price process that the rate of stock price appreciation is $\alpha - \delta = 0.05$ and $\sigma = 0.25$. We are also given $\delta = 0.01$, so $\alpha = 0.06$ and $\phi = 0.06 = (0.06 - r)/0.25$, so $r = \boxed{0.045}$. (D)

62. [Section 17.5] We'll use the obvious generalization of equation (17.10), replacing the subscript 0 with t and the exponent T with $T - t$. Let $a = 2$. Then

$$F_{t,T}(S^2) = S^2 e^{(2(r-\delta)+\sigma^2)(T-t)}$$

Here, $r - \delta = \mu$, so the exponent is $(2\mu + \sigma^2)(T - t)$, which we're given is $0.18(T - t)$. Since $\sigma = 0.4$, it follows that $\mu = \boxed{0.01}$. (A)

63. [Lesson 17] The quadratic variation is defined by

$$V_T^2(U) = \lim_{n \rightarrow \infty} \sum_{i=1}^{[nT]} \left(U(ih) - U(i(h-1)) \right)^2$$

where $h = 1/n$.

For (i) and (iii), we can use the shortcut of d 'ing the process and squaring it and integrating from 0 to 2.4.

For (i), $dW(t) = 2t dt$, and $(2t dt)^2 = 0$ since $(dt)^2 = 0$.

For (iii), $dY(t) = 2dt + 0.9dZ(t)$, and when we square this, since $(dZ(t))^2 = dt$, the square of that is $0.81dt$. Integrating that from 0 to 2.4, we get $2.4(0.81) = 1.944$.

(ii) is not continuous, but it is easy to see that the variations only occur at 1 and 2, regardless of the value of n in the sum defining the quadratic variation, and are 1 apiece, so that the sum of the squared variations is 2. (A)

64. [Section 17.1] The easiest way to work with powers is to log the process. We know that when logging the process, $0.5\sigma^2$ is subtracted from the dt term (see Example 17C).

$$d(\ln Y(t)) = (1.2 - 0.5(0.5^2))dt - 0.5dZ(t) = 1.075dt - 0.5dZ(t)$$

Since $S(t)$ is a stock price, it must be positive, so we take the positive square root of $Y(t)$, or equivalently, $\ln S(t)$ is one-half of $\ln Y(t)$:

$$d(\ln S(t)) = 0.5375dt - 0.25dZ(t)$$

For the upper bound of the 90% lognormal confidence interval, we add 1.645σ to μ , and $\ln S(0) = \ln 8$, so the upper bound of the confidence interval for $\ln S(t)$ is $\ln 8 + 2(0.5375) + 1.645(0.25)\sqrt{2}$. Exponentiating, we get $8e^{2(0.5375)+1.645(0.25)\sqrt{2}} = \boxed{41.93}$. (C)

65. [Section 10.2] It may help to statements (iii)–(vii) in symbols:

(iii) $\alpha - \delta - 0.5\sigma^2 = 0.10$

$$(iv) \quad 2(r - \delta - 0.5\sigma^2) = 0.06, \text{ or } r - \delta - 0.5\sigma^2 = 0.03$$

$$(v) \quad r = 0.04$$

$$(vi) \quad P = 10$$

$$(vii) \quad |S\Delta_{\text{put}}| = 20$$

In (vii), note the use of absolute value signs. The question is intentionally ambiguous, since you are expected to figure out the sign by yourself. When selling a put, you must buy stock to delta hedge, since Δ is negative. So $S\Delta = -20$.

We are being asked for a time-0 rate, not a rate over a period of time, so the concepts of Section 10.2, rather than the concepts of Lesson 5, are appropriate. In that section, the formula on page 207 for the return on an option sets the risk premium of an option equal to elasticity of option times risk premium of stock:

$$\gamma - r = \Omega(\alpha - r) \quad (10.7)$$

We are given enough information to compute the elasticity, which is $S\Delta/P$ (see formula (10.5)):

$$\Omega = \frac{S\Delta}{P} = \frac{-20}{10} = -2$$

and we are given $r = 0.04$. By subtracting (iv) from (iii), we get $\alpha - r = 0.10 - 0.03 = 0.07$. Therefore γ is

$$\gamma = r + \Omega(\alpha - r) = 0.03 - 2(0.07) = -0.10$$

and the absolute value of the rate of return is 10%. (C)

66. [Section 17.3] This question tests your ability to add or subtract $0.5\sigma^2$ when exponentiating or logging a Brownian motion, and your ability to handle negative coefficients for volatility. It is interesting that even in formal exam language, they allow themselves to use a single symbol (like X) for both the stock and its price, although an argument is added to indicate the price.

The Sharpe ratios of X and Y must be equal, since they're based on a single source of uncertainty. The process for $X(t)$ is in ideal form for computing the Sharpe ratio. The rate of return is the stock appreciation rate of 0.06 plus the dividend of 0.02, or 0.08, so the Sharpe ratio of X is

$$\phi_X = \frac{\alpha_X - r}{\sigma_X} = \frac{0.06 + 0.02 - 0.04}{0.2} = 0.2$$

Since Y is given in exponential form, $0.5\sigma^2$ must be added to the coefficient of t to obtain the mean appreciation rate, so $\alpha - \delta = \mu + 0.5(0.1^2) = \mu + 0.005$. We are given $\delta_Y = 0.01$. *The negative sign on the coefficient of $Z(t)$ must be maintained when computing the Sharpe ratio.* Thus

$$\phi_Y = \frac{\alpha_Y - r}{\sigma_Y} = \frac{\mu + 0.005 + 0.01 - 0.04}{-0.1} = \frac{\mu - 0.025}{-0.1}$$

Setting this equal to $\phi_X = 0.2$, we get $\mu - 0.025 = -0.02$ and $\mu = 0.005$. (A)

Note: Incorrect answer (C) is if you forgot the dividend on Y . Incorrect answer (E) is if you used 0.1 instead of -0.1 as the denominator of Y 's Sharpe ratio.

67. [Section 17.3] This question tests your ability to add or subtract $0.5\sigma^2$ when exponentiating or logging a Brownian motion. Since two processes with the same $dZ(t)$ are given, it is obvious that we're going to equate the Sharpe ratios.

The Sharpe ratio for S_2 is obtained by exponentiating the process. In other words, it is necessary to add $0.5\sigma^2$ to the drift of the process for $\ln S_2(t)$. Also, the dividends must be added to the coefficient of dt to obtain the total return. I'll use subscripts of 1 and 2 for the parameters of the S_1 and S_2 respectively.

$$\phi_2 = \frac{\alpha_2 - r}{\sigma_2} = \frac{0.03 + 0.5(0.2^2) + 0.01 - 0.04}{0.2} = 0.1$$

The Sharpe ratio of S_1 is

$$\phi_1 = \frac{\alpha_1 - r}{\sigma_1} = \frac{\mu - 0.04}{20\mu}$$

Equating the two Sharpe ratios,

$$\frac{\mu - 0.04}{20\mu} = 0.1$$

$$\mu - 0.04 = 2\mu$$

$$\mu = \boxed{-0.04} \quad (\text{A})$$

68. [Section 17.6; Table 17.3] This question is a clever way of testing you on the popular Ornstein-Uhlenbeck integral. Using the last two lines of Table 17.3, we see that the stochastic differential equation is for an Ornstein-Uhlenbeck process with $\alpha = 0$, $\lambda = 3$, $\sigma = 2$. Comparing the integral for the process the the solution we are given, we see that $e^{-\lambda t}$ can be factored out of the integral in Table 17.3 to obtain (leaving out the α term, since $\alpha = 0$ in our case)

$$X(t) = X(0)e^{-\lambda t} + e^{-\lambda t} \sigma \int_0^t e^{\lambda s} dZ(s)$$

Thus $A = D = \lambda = 3$, $C = \sigma = 2$, and $A + C + D = 3 + 2 + 3 = \boxed{8}$. (D)

69. [Section 12.3] First we must compute the values of the put option at the nodes. Due to the high interest rate, it is likely that early exercise will be optimal.

The up and down ratios are $u = 1.25$ and $d = 0.8$, as we deduce from the stock values. For example, $150/120 = 1.25$ and $96/120 = 0.8$. Then

$$p^* = \frac{e^{0.1} - 0.8}{1.25 - 0.8} = 0.678158$$

The option pays off at the bottom two nodes at the end, with payments of 24 and 58.56 respectively.

At time 2, the value of the put at the upper node is 0. The middle node is

$$P_{ud} = e^{-0.1}(1 - 0.678158)(24) = 6.989161$$

while at the bottom node,

$$P_{dd}^{\text{tentative}} = e^{-0.1}(0.678158(24) + (1 - 0.678158)(58.56)) = 31.7805$$

but exercise of the option at that node yields $120 - 76.9 = 43.1$, so $P_{dd} = 43.1$.

At time 1, the value of the put at the upper node is

$$P_u = e^{-0.1}(1 - 0.678158)(6.989161) = 2.035349$$

while at the lower node,

$$P_d^{\text{tentative}} = e^{-0.1}(0.678158(6.989161) + (1 - 0.678158)(43.1)) = 16.8401$$

but exercise of the option at that node yields $120 - 96 = 24$, so $P_d = 24$.

At time 0, the value of the put is

$$P = e^{-0.1}(0.678158(2.035349) + (1 - 0.678158)(24)) = 0.238097$$

The tree is shown in Figure B.8.

We are now ready to calculate theta. Since $\varepsilon = 0$ ($\varepsilon = S_{ud} - S_0$, and $S_{ud} = S_0$), it is not necessary to calculate delta and theta, and formula (12.6) simplifies to

$$\theta(S, 0) = \frac{C(Sud, 2h) - C(S, 0)}{2h} = \frac{6.989161 - 8.238097}{2} = \boxed{-0.62447} \quad (\text{A})$$

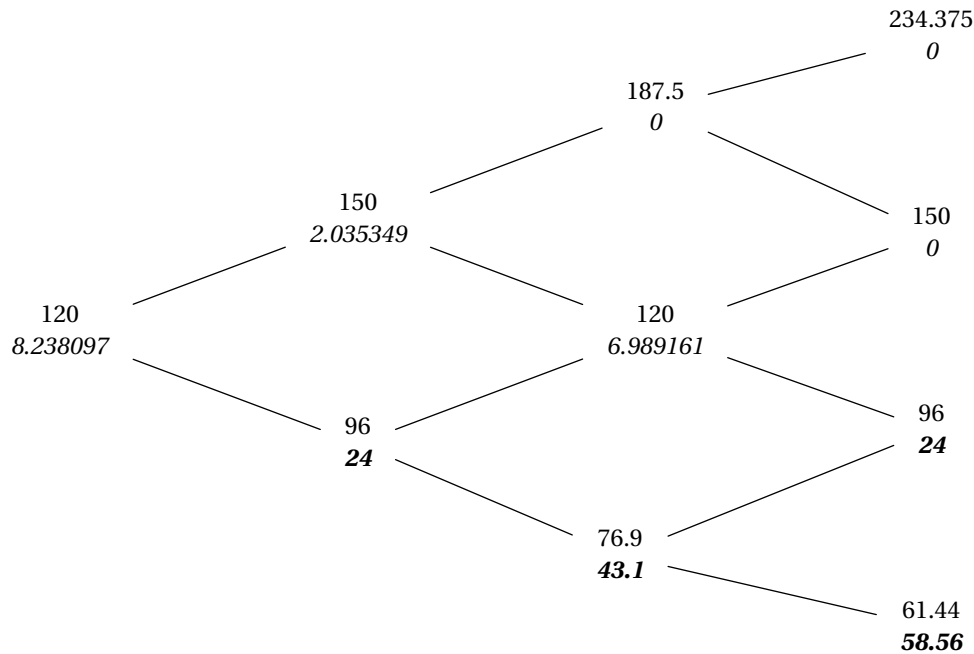


Figure B.8: Binomial tree with put option values for Sample Question 69

70. [Section 17.1 page 385, Section 17.3] The volatility for a proportional portfolio is the proportion times the volatility of the risky asset, or $0.8(0.2) = 0.16$. All the answer choices have this as the volatility.

The Sharpe ratio of the proportional portfolio must equal the Sharpe ratio of the risky asset. The Sharpe ratio of the stock is

$$\phi = \frac{\alpha - r}{\sigma} = \frac{0.1 + 0.02 - 0.05}{0.2} = 0.35$$

Therefore, we back out the α for the proportional portfolio from

$$\begin{aligned} \frac{\alpha_p - 0.05}{0.16} &= 0.35 \\ \alpha_p &= (0.35)(0.16) + 0.05 = 0.106 \end{aligned}$$

To get it into the form of the answer choices, in which an arithmetic Brownian motion is exponentiated, we must subtract $0.5\sigma_p^2$ from α_p : $0.106 - 0.5(0.16^2) = 0.0932$. We see that (C) is the correct answer.

71. [Lesson 17] This question is a general question involving risk-neutral pricing. The basic idea is that the value of the derivative security may be computed using the risk-neutral expected value of the payoff, discounted at the risk-free rate. We will use \mathbf{E}^* to indicate a risk-neutral expectation.

$S(1)$ is a lognormal random variable. The risk-neutral ν of the associated normal random variable is 0.40 (from (iii)). As usual, we subtract $0.5(0.40^2) = 0.08$ from the coefficient of dt to obtain the m parameter of the associated normal random variable; $m = 0.08 - 0.08 = 0$. So $\ln S(1)$ is normal with $m = 0$ and $\nu = 0.4$, or $0.4Z$, where Z is standard normal. Then

$$\mathbf{E}^* \left[1 + S(1) \left(\ln(S(1)) \right)^2 \right] = 1 + \mathbf{E} \left[e^{0.4Z} (0.4Z^2) \right] = 1 + 0.16(1 + 0.4^2)e^{0.08}$$

by the formula provided in (v).

The risk-free rate is the rate earned by the stock in the risk-neutral process. It is 0.08, the coefficient of dt , plus the dividend rate of 0.04, so $r = 0.08 + 0.04 = 0.12$. Therefore, the time-0 price of the derivative security is $e^{-0.12}(1 + 0.16(1.16)e^{0.08}) = \boxed{1.065}$. (C)

72. [Sections 14.2 and 17.5] If you buy a gap call and sell a gap put, you are guaranteed to pay the strike price and receive the asset. (There's an ambiguous case if the ending stock price equals the trigger price, but the probability of that in a continuous process is 0, so we can ignore it.) So the present value of a gap call minus a gap put is the prepaid forward value of the stock minus the strike price, or

$$C - P = F^P(S^2) - Ke^{-rt}$$

where K is the strike price. In our case, $r = 0.07$ and $t = 0.5$. Also, we're given $C - P = 5.543 + 4.745 = 10.288$. So

$$10.288 = F^P(S^2) - 95e^{-0.035} \quad (*)$$

It remains to evaluate $F^P(S^2)$. Use equation (17.11),

$$\begin{aligned} F^P(S^2) &= e^{-rT} S(0)^2 e^{[2(r-\delta)+0.5(2)(1)\sigma^2]T} \\ &= e^{-0.035}(100)e^{[2(0.07-\delta)+0.01](0.5)} \\ &= 100e^{0.04-\delta} \end{aligned}$$

Substituting into (*),

$$\begin{aligned} 100e^{0.04-\delta} - 95e^{-0.035} &= 10.288 \\ e^{0.04-\delta} &= \frac{10.288 + 95e^{-0.035}}{100} = 1.020205 \\ 0.04 - \delta &= \ln 1.020205 = 0.02 \\ \delta &= \boxed{0.02} \quad (\text{A}) \end{aligned}$$

73. [Section 17.5] The Itô process for S^a is given by equation (17.12):

$$\frac{dC}{C} = (a(\alpha - \delta) + 0.5a(a - 1)\sigma^2)dt + a\sigma dZ(t)$$

where $C = S^a$. In our question, $\alpha - \delta = 0.3$ and σ is negated. Equating coefficients of dt and $dZ(t)$ to (ii), we get two equations in a and σ :

$$\begin{aligned} 0.3a + 0.5a(a - 1)\sigma^2 &= -0.66 \\ -a\sigma &= 0.6 \end{aligned}$$

Substituting $\sigma = -0.6/a$ into the first equation, we get

$$\begin{aligned} 0.3a + \frac{0.5(a - 1)(0.6^2)}{a} &= -0.66 \\ 0.3a + 0.18 - \frac{0.18}{a} &= -0.66 \\ 0.3a + 0.84 - \frac{0.18}{a} &= 0 \\ 0.3a^2 + 0.84a - 0.18 &= 0 \end{aligned}$$

Solving the quadratic equation, we get

$$a = \frac{-0.84 \pm \sqrt{0.9216}}{0.6} = 0.2, -3$$

But 0.2 is rejected because it leads to a negative σ . Therefore, $\sigma = -0.6 / -3 = \boxed{2}$. (B)

74. [Section 7.2] We are being asked for the 1st percentile of the payoff of the put option, discounted at 2%. By the structure of the question, we calculate the percentile using true probabilities, but then discount at the risk-free rate, a combination which ordinarily is considered contradictory.

The mean of the associated normal random variable for the 4-year price appreciation is

$$m = (\alpha - 0.5\sigma^2)(t) = (0.1 - 0.5(0.3^2))(4) = 0.22$$

and the standard deviation is

$$v = \sigma\sqrt{t} = 0.3\sqrt{4} = 0.6$$

so the 1st percentile of the variable is $0.22 - 2.326(0.6) = -1.1756$ and the 1st percentile of the stock price is $40e^{-1.1756} = 12.3454$. Then the put pays $40 - 12.3454 = 27.6546$, and discounting this at 2%, $27.6546e^{-0.08} =$

25.53. (E)

75. [Section 15.4] The Boyle modification minimizes variance. As indicated in formula (15.5) on page 343, the resulting variance is the naive variance times the complement of the square of the correlation coefficient, or $25(1 - 0.8^2) =$ **9**. (D)

76. [Section 18.2.2] We need to fill in the missing interest rate at the middle node of the last period. In a BDT tree, the rates for any period follow a geometric progression, so the middle rate is the geometric average of the other two: $r_{ud} = \sqrt{(0.172)(0.106)} = 0.135$, where we round it to three places to be consistent with the official solution. (Without rounding, the final answer would be 0.09 higher.)

We then perform backwards induction. The cap is worthless at the bottom node uu of the last period, and at the other nodes it is worth: (C_x is the value of the cap at node x .)

$$C_{uu} = \frac{10,000(0.172 - 0.115)}{1.172} = 486.348$$

$$C_{ud} = \frac{10,000(0.135 - 0.115)}{1.135} = 176.211$$

At the end of one year, there is a cap payment at the top node but not at the bottom node. When not stated otherwise, a Black-Derman-Toy probability is assumed to have a 0.5 probability of an up move.

$$C_u = \frac{10,000(0.126 - 0.115)(0.5)(486.348 + 176.211)}{1.126} = 391.900$$

$$C_d = \frac{0.5(176.211)}{1.093} = 80.609$$

The price of the cap is

$$C = \frac{0.5(391.900 + 80.609)}{1.09} = \mathbf{216.75} \quad (\text{D})$$

The BDT tree with cap prices is shown in Figure B.9.

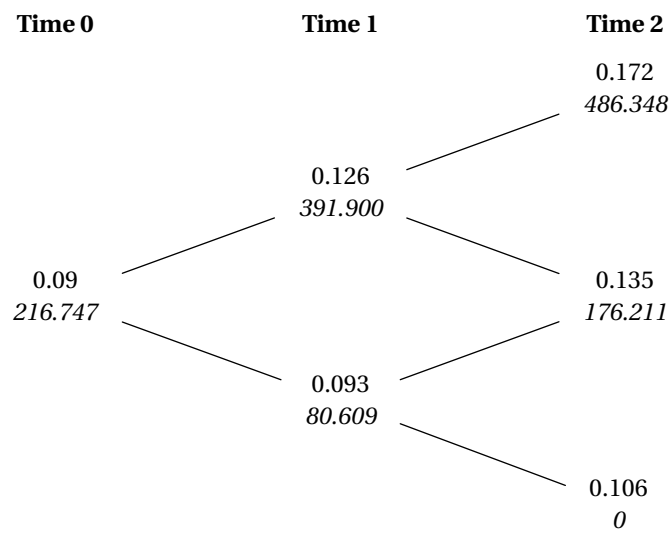


Figure B.9: Black-Derman-Toy tree with cap values for Sample Question 76

Table C.2: Lessons Corresponding to Old and Sample Exam Questions

Sample Questions						MFE-S07		CAS3-S07		CAS3-F07		MFE/3F-S09	
Q	Lesson	Q	Lesson	Q	Lesson	Q	Lesson	Q	Lesson	Q	Lesson	Q	Lesson
1	1	26	2	51	8	1	1	3	1	13	2	1	4
2	2	27	3	52	15	2	5	4	1	14	1	2	13
3	13	28	14	53	14	3	9	12	2	15	1	3	3
4	4	29	18	54	14	4	6	13	1	16	1	4	14
5	4	30	18	55	9	5	10	14	4	17	3	5	18
6	9	31	10	56	13	6	14	15	3	18	4	6	17
7	9	32	17	57	15	7	19	16	3	19	3	7	5
8	10	33	14	58	15	8	9	17	4	20	9	8	17
9	12	34	17	59	15	9	18	20	9	21	9	9	1
10	16	35	17	60	20	10	12	21	9	22	10	10	17
11	16	36	17	61	17	11	4	32	12	23	3	11	17
12	17	37	13	62	17	12	17	33	12	24	12	12	2
13	17	38	20	63	16	13	20	34	13	25	1	13	9
14	20	39	3	64	17	14	3	35	17	26	14	14	18
15	18	40	11	65	10	15	9	36	20	27	13	15	20
16	17	41	14	66	17	17	14	37	18	28	13	16	16
17	6	42	13	67	17	18	17			29	10	17	10
18	14	43	17	68	17	19	12					18	17
19	14	44	4	69	12							19	9
20	10	45	12	70	17							20	12
21	20	46	4	71	17								
22	20	47	12	72	17								
23	16	48	17	73	17								
24	17	49	3	74	7								
25	14	50	7	75	15								
				76	18								

For all exams, only the questions listed above are on the current MFE/3F syllabus