# Errata and updates for ASM Exam 3L (Ninth Edition) sorted by page

[1/24/2010] On page 24, in the solution to exercise 2.4, on the fourth line, change Var($mn$) to Var($mN$).

[2/3/2010] On page 54, in Table 4.1, 4 lines from the bottom, the formula for $\hat{\sigma}$ of a lognormal should have a radical over it: $\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(\ln x_i - \hat{\mu})^2}{n}}$.

[2/3/2010] On page 100, the solution to exercise 6.13 is incorrect. The correct solution is

$$\hat{\beta} = \frac{8100-(30)(450)/15}{270-30^2/15} = 34\tfrac{2}{7}$$
$$\hat{\alpha} = \frac{450}{15} - 34\tfrac{2}{7}\left(\frac{30}{15}\right) = -38\tfrac{5}{7}$$
$$\hat{\varepsilon}_5 = 40 - \left(-38\tfrac{5}{7} + 34\tfrac{2}{7}(3)\right) = \boxed{\mathbf{-24\frac{2}{7}}}$$

[4/9/2010] On page 110, the last 3 lines and final answer to Quiz 7-1 are incorrect. Replace them with:

$$\hat{\beta} = \frac{23.70995-(4.605170)(13.93329)/3}{8.372642-4.605170^2/3} = 1.78109$$
$$\widehat{\ln a} = \frac{13.93329}{3} - 1.78109\left(\frac{4.605170}{3}\right) = 1.91035$$

The estimated value of $Y$ when $X=4$ is $e^{1.91035}\cdot 4^{1.78109} = \boxed{\mathbf{79.795}}$.

[2/3/2010] On page 144, in Table 10.1, 2 lines from the bottom, change $\frac{1}{n}+\frac{1}{n}$ to $\frac{1}{m}+\frac{1}{n}$.

[2/3/2010] On page 174, in the solution to exercise 12.18, the paragraph beginning on the second line of the page is incorrect. Replace it with

$S_2^2/S_1^2$ is $\sigma_2^2/\sigma_1^2$ times an $F(11,24)$ random variable, whose 95th percentile is 2.22 and whose 99th percentile is 3.09. Under the null hypothesis $\sigma_2^2/\sigma_1^2 = 0.5$, so the 99th percentile of $\sigma_2^2/\sigma_1^2$ times the $F(11,24)$ random variable is 1.55. Since $S_2^2/S_1^2 = 1.75/0.75 = 4.67$, the null null hypothesis is rejected even at 1%, and certainly at 5%, significance.

[1/29/2010] On page 174, the second paragraph of the solution to exercise 12.19 uses the wrong value of $F(14,9)$. Change 2.98 to 3.02 in the two places it occurs. Change 0.3883 to 0.3832 in the two places it occurs. Change 2.575 to 2.610 in the two places it occurs; the final answer is $\boxed{\mathbf{(0.326, 2.610)}}$.

[2/3/2010] On page 181, in Table 13.1, 2 lines from the bottom, an = is missing between $T_{N-2}$ and $\sqrt{F_{1,N-2}}$.

[3/25/2010] On page 208, in the solution to exercise 15.7, on the fourth displayed line, change $n\sum X_i$ to $n+\sum X_i$.

[2/3/2010] On page 214, in exercise 16.7, on the first line, change "a uniform" to "an exponential". (The question is trivial for a uniform distribution.)

[2/3/2010] On page 216, the solution to exercise 16.10 is incorrect. The correct solution is

The distribution function of $X$ is $F_X(x) = 1-(1-x)^2$, $0 \le x \le 1$. The density function of the 3rd order statistic is

$$f_{Y_3}(y) = \frac{4!}{2!}\left(1-(1-y)^2\right)^2\left(2(1-y)\right)\left(1-y\right)^2 = 24\left((1-y)^3 - 2(1-y)^5 + (1-y)^7\right)$$

Integrating this from 0 to 1:

$$\mathbf{E}[Y_3] = 24\left(\int_0^1 \Big(y(1-y)^3 - 2y(1-y)^5 + (1-y)^7\Big)\,\mathrm{d}y\right)$$

$y(1-y)^3$ is the integrand of a beta with $a=2$, $b=4$, so the integral is $\Gamma(2)\Gamma(4)/\Gamma(6) = 3!/5! = 1/20$. $y(1-y)^5$ is the integrand of a beta with $a=2$, $b=6$, so the integral is $\Gamma(2)\Gamma(6)/\Gamma(8) = 5!/7! = 1/42$. $y(1-y)^7$ is the integrand of a beta with $a=2$, $b=8$, so the integral is $\Gamma(2)\Gamma(8)/\Gamma(10) = 7!/9! = 1/72$. Thus

$$\mathbf{E}[Y_3] = 24\left(\frac{1}{20} - \frac{1}{21} + \frac{1}{72}\right) = \boxed{\mathbf{0.390476}}$$

[12/22/2009] On page 225, in Table 17.1, on the first two lines of "Mathematical Probability Functions" on the right hand side, delete the extra $(x)$ in the subscript and add an argument $(t)$, so that you get $s_{T(x)}(t)$ and $F_{T(x)}(t)$. On the last line, change the numerator of the fraction to $F_X(x+t+u) - F_X(x+t)$, and in the denominator make the $x$ in parentheses lowercase.

[12/4/2009] On page 228, in the table for question 17.7, the 0.07 in the last column should be on the line for $x = 62$. Also, the 70 in the $d_x$ column, row for $x = 62$, should be deleted, although it is correct, since the intention of the question was for the student to derive that value.

[12/4/2009] On page 231, the solution to exercise 17.7 is incomplete. The correct solution is

Did you notice that you are given ${}_{x-60|}q_{60}$ rather than $q_x$?

Since ${}_{2|}q_{60} = 0.07$, then $d_{62} = 0.07 l_{60} = 70$ and $l_{62} = l_{63} + d_{62} = 780 + 70 = 850$. Then $l_{61} = 850 + d_{61} = 950$ and $d_{60} = l_{60} - l_{61} = 1000 - 950 = 50$, so $q_{60} = d_{60}/l_{60} = 50/1000 = \boxed{\mathbf{0.05}}$.

[12/2/2009] On page 235, in formula (18.1), there should be an "ln" before the last $s_X(x)$:

$$\mu_x = \frac{f_X(x)}{s_X(x)} = -\frac{\mathrm{d}}{\mathrm{d}x}\ln s_X(x)$$

[1/24/2010] On page 236, two lines above Quiz 18-1, change the denominator $65+60$ to $65+70$.

[1/10/2010] On page 245, in exercise 18.30, change ${}_t p_x$ to $p_x$ once apiece on lines 1 and 3, and change $s \le t$ to $s \le 1$ once apiece on lines 1 and 2.

[1/10/2010] On page 252, in the solution to exercise 18.30, on the displayed line, change $e^{-0.2}$ to $e^{0.05}$.

[12/11/2009] On page 256, the second displayed equation in Subsection 19.1.2 is missing an $x$ and should be

$$s_{T(x)}(t) = \frac{\omega - x - t}{\omega - x}$$

[2/21/2010] On page 413, third line of answer to Example 26A, change $T < -(\ln 0.25)/\delta$ to $T > -(\ln 0.25)/\delta$.

[1/24/2010] On page 257, on the second line, change $\omega = 70$ to $\omega - x = 70$.

[2/3/2010] On page 257, 5 lines from the bottom of the page, replace the formula with

$$f_{T(x)}(t) = \frac{\alpha(\omega - x - t)^{\alpha-1}}{(\omega - x)^{\alpha}}$$

[2/8/2010] On page 257, on the last line of the page, add an $\alpha$ to the numerator:

$$\mathrm{Var}\Big(T(x)\Big) = \frac{\alpha(\omega - x)^2}{(1+\alpha)^2(2+\alpha)}$$

[1/15/2010] On page 268, on the first two displayed lines of the answer to Example 20A, replace $\mathrm{d}t$ with $\mathrm{d}u$.

[12/14/2009] On page 269, on the second line, change $\mathring{e}_{30:\overline{40}|}$ to $\mathring{e}_{30:\overline{25}|}$.

[12/15/2009] On page 269, on the second displayed line of Subsection 20.1.2, the left hand side should be $\mathbf{E}\left[\left(T(x)\wedge n\right)^2\right]$. Three lines further down, replace the final result with $\dfrac{1-(1+\mu n)e^{-\mu n}}{\mu^2}$. On the next line, replace the equation with $\mathrm{Var}\left(T(x)\wedge n\right)=\mathbf{E}\left[\left(T(x)\wedge n\right)^2\right]-\mathring{e}^2_{x:\overline{n}|}$.

[3/1/2010] On page 269, on the last line, add an $\alpha$ to the numerator:

$$\mathrm{Var}\left(T(x)\right)=\frac{\alpha(\omega-x)^2}{(\alpha+1)^2(\alpha+2)}$$

[12/23/2009] On page 271, on the last line of the answer to Example 20E, change the first $\dfrac{3}{8}$ to $\dfrac{5}{8}$ and change the final answer to 276.692708.

[1/5/2010] On page 274, in the answer to Example 20F, on lines 3–5, the left hand sides should be ${}_2p_{90}$, ${}_3p_{90}$, and ${}_4p_{90}$ respectively.

[12/20/2009] On page 275, in the answer to Example 20G, change 99.50008 to 99.5008.

[1/20/2010] On page 275, on the first line of the answer to Example 20H part 3, change (20.12) to (20.13).

[12/20/2009] On page 284, in the solution to exercise 20.4, on the first line, change the reference to formula (20.16) to (20.15).

[12/20/2009] On page 284, in the solution to exercise 20.10, on the second and third lines, change the second denominators to $2(\omega-20)$.

[1/24/2010] On page 287, the solution to exercise 20.21 is incorrect. The correct solution is

For this deMoivre, we have

$$\begin{aligned}
e_{10:\overline{20}|} &= \mathring{e}_{10:\overline{20}|}-0.5\,{}_{20}q_{10}\\
18 &= 20\,{}_{20}p_{10}+9.5{}_{20}q_{10}\\
&= 20-10.5\,{}_{20}q_{10}\\
{}_{20}q_{10} &= \frac{2}{10.5}=\frac{4}{21}\\
\frac{20}{\omega-10} &= \frac{4}{21}\\
\omega &= \boxed{\mathbf{115}}
\end{aligned}$$

[12/20/2009] On page 291, on the second displayed line of the solution to Quiz 20-3, change the left hand side to $\mathbf{E}\left[\left(K(20)\wedge 3\right)^2\right]$

[12/24/2009] On page 292, on the second displayed line of the solution to Quiz 20-4, put a presubscript of 24 before the last $q_{20}$: $11.5{}_{24}q_{20}$.

[1/27/2010] On page 298, in Table 21.1, formula (21.4), replace the subscript $x+k$ with $x+1$.

[2/1/2010] On page 309, in the solution to exercise 21.17, replace the fourth line with

$${}_tp_{40}=\exp\left(-\int_0^t\frac{\mathrm{d}u}{k-(40+u)}\right)$$

[1/20/2010] On page 337, in the second displayed equation the left hand side should be $\ddot{a}_{\overline{n}|}$, and in the third displayed equation the left hand side should be $\bar{a}_{\overline{n}|}$.

[2/24/2010] On page 345, the first six lines of the page are not well stated. Replace them with

For the deferred insurance, we will use formula

$$\bar{A}_x = {}_xE_x\,A_{x+n}$$

First we calculate the 5-year pure endowment at $\delta$ and $2\delta$; $\mu = 0.01$ in this period.

$$\begin{aligned} {}_5E_x &= e^{-5(0.01+0.06)} = 0.704688 \\ {}^2_5E_x &= e^{-5(0.01+0.12)} = 0.522046 \end{aligned}$$

Then we calculate $A_{x+5}$ at $\delta$ and $2\delta$; $\mu = 0.02$ in this period.

$$\begin{aligned} \bar{A}_{x+5} &= \frac{\mu}{\mu+\delta} = \frac{0.02}{0.02+0.06} = 0.25 \\ {}^2\bar{A}_{x+5} &= \frac{\mu}{\mu+2\delta} = \frac{0.02}{0.02+0.12} = \frac{1}{7} \end{aligned}$$

[12/9/2009] On page 347, on the first line of the page, remove the bar from $\bar{A}$.

[1/9/2010] On page 357, the solution to exercise 23.12 is incorrect. The correct solution is

The 30-year term insurance has $\delta = \ln(1+i) = \ln 1.04$, so its expected present value is

$$\frac{0.02}{0.02+\ln 1.04}(1-e^{-30(0.02+\ln 1.04)}) = 0.280575$$

The pure endowment paid when $T \geq 30$ has present value 0.5 if paid and expected present value $0.5e^{-30(0.02)} = 0.274406$. The total expected value of $Z$ is $\mathbf{E}[Z] = 0.280575 + 0.274406 = \boxed{\mathbf{0.55498}}$.

[1/3/2010] On page 357, in the solution to exercise 23.13, on the first displayed line, replace ${}_kE_{x+1}$ with ${}_{k+1}E_x$. Also, a 2 is missing from the right hand side, which should be $\frac{2e^{-0.06k}}{3}\left(1-e^{-0.06}\right)$. The other 3 displayed lines should be replaced with

$$\begin{aligned} \bar{A} &= 1000\left(\frac{2(1-e^{-0.06})}{3}\right)\sum_{k=0}^{\infty} e^{-0.06(2k+1)} \\ &= 38.8236\left(\frac{e^{-0.06}}{1-e^{-0.12}}\right) \\ &= 38.8236(8.328335) = \boxed{\mathbf{323.34}} \end{aligned}$$

[12/31/2009] On page 362, in the solution to Quiz 23-2, the last line should be

$$\bar{A} = 10\left(\frac{0.02}{0.01}\right) = \boxed{\mathbf{20}}$$

[12/21/2009] On page 365, in Example 24C(i), change $1/(50-t)$ to $2/(50-t)$.

[2/1/2010] On page 369, in Table 24.2, formulas (24.4) and (24.5), replace every $n$ (one in (24.4), three in (24.5)) with $u$.

[1/21/2010] On page 376, in the solution to exercise 24.5, on the second line, delete $\int_0^{60} e^{-0.06t}\,dt$.

[1/21/2010] On page 377, in the solution to exercise 24.10, replace $\bar{a}_{\overline{75}|}$ with $\bar{a}_{\overline{25}|}$ on the first and fourth displayed lines.

[12/15/2009] On page 381, the solution to Quiz 24-1 is incorrect. Replace the three displayed lines with

$$\bar{A}^{1}_{30:\overline{20}|} = \frac{\bar{a}_{\overline{20}|}}{90} = \frac{1-e^{-0.04(20)}}{90(0.04)} = 0.152964$$

$$\bar{A}_{30:\overline{20}|^{1}} = e^{-0.04(20)}\left(\frac{70}{90}\right) = 0.349478$$

$$\bar{A}_{30:\overline{20}|} = 0.152964 + 0.349478 = \boxed{\mathbf{0.5024}}$$

[1/3/2010] On page 382, the first line of the page is incorrect, resulting in the other lines being incorrect. The solution is unnecessarily complicated. A replacement page is provided at the end of these errata.

[2/9/2010] On page 384, two lines below formula (25.3), change $\mathbf{E}[Z]$ to $\mathbf{E}[Z^2]$.

[2/9/2010] On page 384, on the third line of the answer to Example 25B, change "endowment insurance" to "pure endowment".

[12/30/2009] On page 386, 6 lines from the bottom, change "force of interest" to "interest factor".

[12/30/2009] On page 387, on the 4th line, capitalize the "y" in "you".

[2/9/2010] On page 407, in the solution to exercise 25.33, on the second line from the end, put a minus sign before $b/2a$.

[3/12/2010] On page 409, in the solution to exercise 25.41, on the second displayed line, change the presubscript of $q$ to $0.25k|0.25$.

[1/7/2010] On page 410, in the last three lines of Quiz 25-1:

- Replace $e^{-0.02}$ with $e^{-0.2}$
- Replace the second to last line with $(0.457175)(0.315237) = 0.144119$.
- On the last line, replace 0.172541 with 0.144119 and 307.23 with 278.81.

The single benefit premium for the insurance is $1000A_{60} = 1000(0.134693 + 0.144119) = \boxed{\mathbf{278.81}}$.

[3/5/2010] On page 410, in the solution to Quiz 25-2, remove the four bars from the $A$'s.

[3/6/2010] On page 416, on the second line, replace "$p$th percentile of $Z$" with "$100p$th percentile of $Z$".

[1/31/2010] On page 416, 4 lines above Example 26G, change $\Pr(Z > z)$ to $\Pr(Z < z)$.

[1/10/2010] On page 418, in Quiz 26-3, on the first line, change "at the end of the year" to "at the moment of". For discrete insurances, the percentile is not well-defined, and while the answer given in the manual is correct, it is not the only answer.

[1/25/2010] On page 422, in question 26.20, delete the word "percentile" on the last line.

[12/18/2009] On page 424, the solution to exercise 26.7 is incorrect. The correct solution is

If $v^T = 0.5$, then $T = (\ln 0.5)/(\ln v) = -(\ln 0.5)/(\ln 1.06) = 11.8957$. Thus $Z > 500$ if and only if $K + 1 \le 11$, which means death occurs within the first 11 years. Under deMoivre, the probability of this is ${}_{11}q_{20} = 11/80 = \boxed{\mathbf{0.1375}}$.

[12/18/2009] On page 424, the solution to exercise 26.8 is incorrect. The correct solution is

If $v^T = 0.6$, then $T = (\ln 0.6)/(\ln v) = -(\ln 0.6)/(\ln 1.06) = 8.7667$. Thus $Z \le 600$ if and only if $K+1 \ge 9$, which means survival for 8 years. Using the ILT, ${}_8p_{45} = l_{53}/l_{45} = 8{,}779{,}128/9{,}164{,}051 =$ **0.95800**. Notice that the answer would've been the same for a whole life policy or a 20-year term policy.

[12/18/2009] On page 433, Example 27C is defective and the answer is incorrect. Change (i) to $A_{85} - A_{65} = 0.15$. Change the 6 displayed lines of the answer to

$$\begin{aligned} A_{65} &= A^{1}_{65:\overline{20|}} + {}_{20}E_{65}\,A_{85} \\ &= A_{65:\overline{20|}} - {}_{20}E_{65} + {}_{20}E_{65}(A_{65} + 0.15) \\ &= 0.7 + {}_{20}E_{65}(-0.85 + A_{65}) \\ &= 0.7 + 0.5(-0.85 + A_{65}) \\ 0.5A_{65} &= 0.275 \\ A_{65} &= \boxed{\mathbf{0.55}} \end{aligned}$$

[2/23/2010] On page 435, the denominator of equation (27.6) should be $(\mu+2\delta)^3$.

[3/12/2010] On page 435, 2 lines from the bottom, $\bar{A}_{x\overline{10|}}$ should be $\bar{A}^{1}_{x:\overline{10|}}$..

[2/1/2010] On page 440, in Table 27.1, formulas (24.4) and (24.5), replace every $n$ (one in (24.4), three in (24.5)) with $u$.

[2/23/2010] On page 440, the denominator of equation (27.6) should be $(\mu+2\delta)^3$.

[1/11/2010] On page 445, in question 27.19, on the first line, add "at the moment of death" after the word "death".

[2/13/2010] On page 454, in the solution to exercise 14.21, the final answer should be multiplied by 1000, and is **1160**.

[12/18/2009] On page 456, the solution to Quiz 27-1 is incorrect. The correct solution is

By the recursive formula,

$$\begin{aligned} 0.12966 &= 0.006v + (0.994v)(0.13032) \\ &= 0.135538v \\ 1+i &= \frac{0.135538}{0.12966} = 1.04533 \\ i &= \boxed{\mathbf{0.0453}} \end{aligned}$$

[1/9/2010] On page 457, the final answer to Quiz 27-3 should be 0.9771 instead of 0.8926. A simpler way to solve the quiz, after getting $(IA)^{1}_{40:\overline{20|}} = 0.98$, is

Use formula (27.11). $(IAA)^{1}_{41:\overline{19|}}$ is the same for both mortality tables. We'll use primes for the modified mortality table. We have

$$(IAA)^{1}_{41:\overline{19|}} = \frac{0.98 - 0.005(0.95)}{0.995v} = \frac{0.980151}{v}$$

and

$$(IA)^{1}_{40:\overline{20|}} = 0.9v\left(\frac{0.980151}{v}\right) + 0.1(0.95) = \boxed{\mathbf{0.9771}}$$

[2/21/2010] On page 460, in Table 28.1, "Deferred life annuity" line, "Present value" column, change $t$ to $T$.

[2/21/2010] On page 461, on the fifth line, replace $a_{\overline{I}|}$ with $\bar{a}_{\overline{I}|}$.

[2/13/2010] On page 464, on the first displayed line of the answer to Example 28D, remove $\delta$ from the denominator.

[2/21/2010] On page 470, in exercise 28.15, on the first line, delete the second "You are given".

[3/15/2010] On page 472, in the solution to exercise 28.3, on the first line, put a bar on $A_{40:\overline{10}|}$.

[3/16/2010] On page 473, in the solution to exercise 28.9, on the third displayed line, change the presubscript from 5 to 5|.

[12/5/2009] On page 474, in the solution to exercise 28.11, on the 6th line, change equatio to equation.

[12/21/2009] On page 475, in the solution to exercise 28.18, multiply the final answer by 10, or add the line

$$\bar{a}_{30:\overline{20}|} = \boxed{\mathbf{90.484}}$$

[12/22/2009] On page 479, in Example 27A, the last column of the table should be headed $\ddot{a}_{\overline{k+1}|}$. The answer is incorrect. Here is an answer that uses an improved version of equation (29.5):

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=1}^{n-1} \ddot{a}_{\overline{k}|}\, {}_{k-1}p_x\, q_{x+k-1} + \ddot{a}_{\overline{n}|}\, {}_{n-1}p_x$$

The probabilities of death in each of the first two years is given in the second column of the table, since ${}_{k|}q_x = {}_k p_x\, q_{x+k}$. The probability of survival for 2 years is ${}_2p_x = 1-0.10-0.15-0.75$. Using formula (29.5), the expected present value of the 3-year temporary life annuity is

$$\ddot{a}_{x:\overline{3}|} = 0.10(1)+0.15(1.92)+0.75(2.75) = \boxed{\mathbf{2.4505}}$$

[1/24/2010] On page 481, 4 lines from the bottom, move the diuresis from the second $a$ to the first: $\ddot{a}_{x:\overline{n}|} = a_{x:\overline{n}|} + 1 - {}_nE_x$.

[3/19/2010] On page 484, on the first line of the answer to Example 29F, change (29.1) to (29.3).

[4/3/2010] On page 485, in the fourth timeline of the page, the one for $s_{x+1:\overline{n-1}|}$, add $x+$ before the last four expressions $n-2$, $n-1$, $n$ and $n+1$:


[2/21/2010] On page 486, on the first line of the answer to Example 29I, change ${}_{10}E_{55}$ to ${}_{10}E_{45}$.

[2/21/2010] On page 487, once apiece on the two displayed lines of the answer to Example 29J, change ${}_{10}E_{65}$ to ${}_{10}E_{55}$.

[12/23/2009] On page 496, in exercise 29.24, the three annuities of (i), (ii), and (iii) should all be due; put double-dots on top of each $a$.

[1/11/2010] On page 500, in the solution to exercise 32.20, on the third line from the end, replace 455.067 with 349.403. On the last line, replace 465.067 with 359.403 and 40.88 with 31.59.

[12/23/2009] On page 505, in the solution to exercise 29.24, interchange the two $a$'s on the first displayed line:

$$a_{x:\overline{30}|} = \ddot{a}_{x:\overline{31}|} - 1 = 17$$

[1/11/2010] On page 506, the solution to exercise 29.29 is incorrect. The correct solution is

The annuity is like an 11-year deferred annuity due except that it also pays $\ddot{a}_{\overline{11}|}$ if (45) survives 10 years. Therefore

$$\begin{aligned}\mathbf{E}[Y] &= {}_{11}E_{45}\,\ddot{a}_{56} + {}_{11}p_{45}\,\ddot{a}_{\overline{11}|} \\ &= \left(\frac{0.52652}{1.06}\right)\left(\frac{8{,}563{,}435}{8{,}640{,}861}\right)(12.0604) + \left(\frac{8{,}563{,}435}{9{,}164{,}051}\right)\left(\frac{1-(1/1.06^{11})}{0.06/1.06}\right) \\ &= 5.93693 + (0.934460)(8.360087) = \boxed{\mathbf{13.75}}\end{aligned}$$

[2/21/2010] On page 507, in the solution to exercise 17.32, on the first displayed line, change $\mathbf{E}[Z]$ to $\mathbf{E}[Y]$.

[2/12/2010] On the last two lines of page 511, the + before $\delta^2$ should be − and the + in equation (18.6) should be −:

$$\begin{aligned}&= \frac{1-2\delta\,{}^2\bar{a}_{x:\overline{n}|} - 1 + 2\delta\,\bar{a}_{x:\overline{n}|} - \delta^2(\bar{a}_{x:\overline{n}|})^2}{\delta^2} \\ &= \frac{2(\bar{a}_{x:\overline{n}|} - {}^2\bar{a}_{x:\overline{n}|})}{\delta} - (\bar{a}_{x:\overline{n}|})^2\end{aligned}$$

[12/24/2009] On page 516, in Quiz 30-2, add the words "the variance of" after the word "Calculate".

[2/12/2010] On page 518, in Table 30.1, in equation (30.6), change the + sign to a −:

$$\frac{2(\bar{a}_{x:\overline{n}|} - {}^2\bar{a}_{x:\overline{n}|})}{\delta} - \bar{a}^2_{x:\overline{n}|}$$

[1/11/2010] On page 525, in exercise 30.31, at the end of the first line, delete the s in deaths.

[11/18/2009] On page 525, in the solution to exercise 30.1, in the second bullet, change $E[T(x)]^2$ to $\mathbf{E}[T(x)^2]$.

[4/9/2010] On page 526, in the solution to exercise 30.4, on the third displayed line, add $\mu$ before the second slash.

[1/29/2010] On page 532, the final answer to exercise 30.23 should be $\boxed{\mathbf{0.23503}}$.

[4/9/2010] On page 533, in the solution to exercise 30.27, change "is" to "if".

[12/24/2009] On page 535, the solutions to Quizzes 30-2 and 30-3 are incorrect. A replacement page is provided at the end of the errata.

[2/21/2010] On page 539, change the caption of Figure 31.3(d) to "10 year certain & life annuity".

[4/9/2010] On page 540, on the first line of the answer to Example 31B, change 20 to 16.

[2/21/2010] On page 541, in Quiz 31-1, on the first line, change (40) to (70).

[11/18/2009] On page 541, 1 line above Example 31E, change "are greater" to "is greater".

[2/21/2010] On page 541, in the answer to Example 31E, on the first displayed line, remove the bar from $\bar{A}_{55}$.

[12/25/2009] On page 542, replace the last two lines of the answer to Example 31F with

The value of the annuity-certain paying 2 per year for 25 years and 1 per year from $t = 25$ to $t = 35.09925$ is

$$\frac{1-e^{-0.05(25)}}{0.05} + \frac{1-e^{-0.05(35.09925)}}{0.05} = 14.2699 + 16.5417 = \boxed{\mathbf{30.8116}}$$

[3/27/2010] On page 545, replace the 3 lines after the answer to Example 31I with

A whole life annuity on $x$ may be split into an $n$-year temporary annuity plus an $n$-year pure endowment factor times an annuity on $x+n$.

$$\ddot{a}_x = \ddot{a}_{x:\overline{n}|} + {}_nE_x\,\ddot{a}_{x+n}$$

[12/5/2009] On page 555, in the solution to exercise 31.4, on the 6th displayed line, change $t$ to $T$.

[1/11/2010] On page 556, the solution for exercise 31.8 is for a 20-year deferred annuity, but the question asks for a 10-year deferred annuity. The correct solution is:

The actuarial present value of the deferred annuity is, by the current payment formula with $v^k = 1$,

$$\sum_{k=10}^{\infty} e^{-0.01k} = \frac{e^{-0.1}}{1-e^{-0.01}} = 90.937$$

The probability of more than 90 payments is the probability of surviving 100 years, or $e^{-0.01(100)} = \boxed{\mathbf{0.3678}}$.

[4/21/2010] On page 556, in the solution to exercise 31.12, change the left hand side of the second displayed line from ${}_{3|}\bar{a}_t$ to ${}_{3|}\bar{a}_{\overline{t-3}|}$.

[4/11/2010] On page 557, on the last line of the solution to exercise 31.15, change the left-hand side to ${}_{24.3642}p_{35}$.

[4/11/2010] On page 558, on the second line of the solution to exercise 31.21, change 0.5 to 0.4.

[2/21/2010] On page 562, on the first displayed line of the solution to Quiz 31-1, change the subscript $\overline{T}|$ to $\overline{t}|$.

[1/9/2010] On page 563, in the solution to Quiz 31-3, change both 60's to 20's. While the 25th percentile of age at death is 60, the 25th percentile of time to death is 20.

[4/15/2010] On page 566, two lines above the last displayed line on the page, delete "a" and change "aer" to "are".

[2/23/2010] On page 569:

- In the answer to Example 32E, first displayed line, change $\bar{a}_{40}$ to $\bar{a}_{\overline{10}|}$.
- In the answer to Example 32F, second displayed line, change $\mu+\pi$ to $\mu-\pi$

[12/30/2009] On page 574, exercise 32.17 requires the formula relating insurances paid at the end of the year of death to those paid at the moment of death, which is not on the CAS syllabus, so skip the exercise.

[12/28/2009] On page 578, in the solution to exercise 32.15, on the second displayed line, replace $\overline{n}|$ with $\overline{20}|$. Replace the last two lines of the solution with

Then $A_{x:\overline{20}|}^{\ 1} = 0.40594 - 0.15 = 0.25594 = {}_{20}p_x\, e^{-20(0.06)}$, so

$${}_{20}p_x = 0.25594e^{1.2} = \boxed{\mathbf{0.8498}}$$

[2/20/2010] On page 580, change double-dot to bar on top of the $a$'s on line 2 of the solution to Quiz 32-1 and lines 3 and 6 of the solution to Quiz 32-2. On line 3 of the solution to Quiz 21-2, switch $\bar{A}_{50:\overline{20}|}$ and $\bar{a}_{50:\overline{20}|}$: $\bar{A}_{50:\overline{20}|} = 1 - \delta\bar{a}_{50:\overline{20}|}$.

[2/22/2010] On page 584, on the fourth and fifth lines, change ${}_{k-1}q_{45}$ to ${}_{k-1|}q_{45}$.

[1/10/2010] On page 584, in Quiz 33-3, on the second line, change 4% to 6%. In (iii), change $x \le 65$ to $x < 65$ and $x > 65$ to $x \ge 65$.

[1/27/2010] On page 587, in Table 33.1, on the line after **Constant Mortality**, change $P_{x:\overline{n}|} = vq_x$ to $P^{1}_{x:\overline{n}|} = vq_x$.

[4/7/2010] On page 604, in the solution to exercise 33.3, on the first displayed line, ${}_{k-1}p_x$ is missing. It should read

$$A = \sum_{k=1}^{3} b_k\, {}_{k-1}p_x\, q_{x+k-1}\, v^k$$

[4/3/2010] On page 614, the last four lines of the solution to exercise 33.38 are incorrect. Replace them with:

$$(IA)^{1}_{20:\overline{10}|} = \frac{1}{80}\sum_{k=1}^{10} k v^k = \frac{(Ia)_{\overline{10}|}}{80} = \frac{\ddot{a}_{\overline{10}|} - 10/1.05^{10}}{80(0.05)} = \frac{8.107822 - 6.139133}{4} = 0.492172$$

Substituting into (*),

$$7.692389\pi = 0.492172\pi + 8163.541$$
$$\pi = \frac{8163.541}{7.692389 - 0.492172} = \boxed{\mathbf{1133.79}}$$

[1/11/2010] On page 614, in the solution to exercise 33.39, replace the fourth displayed line with

$$a_{20:\overline{10}|} = \sum_{k=0}^{9}\left(\frac{0.99}{1.05}\right)^k = \frac{1-(0.99/1.05)^{10}}{1-(0.99/1.05)}$$
$$\ddot{s}_{20:\overline{10}|} = \left(\frac{1.05}{0.99}\right)^{10}\ddot{a}_{s:\overline{10}|} = \left(\frac{1.05}{0.99}\right)^{10}\left(\frac{1-(0.99/1.05)^{10}}{1-0.99/1.05}\right) = 14.01948$$

and the last line with

$$\pi = \frac{17{,}500}{0.5(13.20679 + 14.01948)} = \boxed{\mathbf{1285.52}}$$

[11/18/2009] On page 617, in the solution to exercise 33.50, on the fourth line, change $\frac{1}{\ddot{a}_{20}}$ to $\ddot{a}_{20}$.

[12/29/2009] On page 619, in the solution to exercise 33.57, on the second displayed line, change 0.04 to 0.02. Replace the last two lines with

$$0.06 - {}_nP_x = 0.05(1-0.4) = 0.03$$
$${}_nP_x = 0.06 - 0.03 = \boxed{\mathbf{0.03}}$$

[1/10/2010] With the corrections to Quiz 33-3 mentioned above, the printed solution is correct. Without the corrections, the question is much more difficult than a typical exam question. The following is the solution to Quiz 33-3 as stated in the manual before the corrections:

Let $P$ be the annual benefit premium for an insurance of 1. By the equivalence principle,

$$P\ddot{a}_{35} = {}_{30|}A_{35} + P\sum_{k=1}^{30}\ddot{s}_{\overline{k}|0.04}(1.06^{-k})\,{}_{k-1|}q_{35}$$

We must evaluate ${}_{30|}A_{35}$, $\ddot{a}_{35}$, and the sum. Since $A_{66} = q/(q+i) = 2/7$, we have

$${}_{30|}A_{35} = \frac{0.99^{30}}{1.06^{30}}\left(\frac{0.01}{1.05} + \frac{0.99}{1.05}\left(\frac{2}{7}\right)\right) = 0.0359208$$

To evaluate $\ddot{a}_{35}$, we split up the different interest and mortality rate periods.

$$\ddot{a}_{35} = \sum_{k=0}^{30}\left(\frac{0.99}{1.06}\right)^k + \left(\frac{0.99}{1.06}\right)^{30}\left(\frac{0.99}{1.05}\right)\sum_{k=0}^{\infty}\left(\frac{0.98}{1.05}\right)^k$$

$$= \frac{1-(0.99/1.06)^{31}}{1-0.99/1.06} + \left(\frac{0.99}{1.06}\right)^{30}\left(\frac{0.99}{1.05}\right)\left(\frac{1}{1-0.98/1.05}\right)$$
$$= 13.32141 + 1.82145 = 15.14286$$

The sum is evaluated as follows:

$$\sum_{k=1}^{30} \ddot{s}_{\overline{k}|0.04}(1.06^{-k})_{k-1|}q_{35} = \sum_{k=1}^{30}\left(\frac{1-(1/1.04)^k}{0.04/1.04}\right)\left(\frac{1.04}{1.06}\right)^k (0.01)(0.99^{k-1})$$
$$= \left(\frac{0.01}{0.99}\right)\left(\frac{1.04}{0.04}\right)\sum_{k=1}^{30}\left(\left(\frac{(1.04)(0.99)}{1.06}\right)^k - \left(\frac{0.99}{1.06}\right)^k\right)$$
$$= \left(\frac{0.01}{0.99}\right)\left(\frac{1.04}{0.04}\right)\sum_{k=1}^{30}\left(0.971321^k - 0.933962^k\right)$$
$$= \left(\frac{0.01}{0.99}\right)\left(\frac{1.04}{0.04}\right)\left(\frac{0.971321-0.971321^{31}}{1-0.971321} - \frac{0.933962-0.933962^{31}}{1-0.933962}\right)$$
$$= 1.94335$$

The premium for the deferred insurance of 1000 is $P = 1000(0.0359208)/(15.14286-1.94335) = \boxed{\mathbf{3.157}}$.

[12/26/2009] On page 623, in the first sentence of Section 34.1, delete the first "is". At the beginning of the sixth line, change "of issue" to "at issue".

[12/30/2009] On page 629, in exercise 34.1(i), add at the end "with $\omega = 100$".

[2/22/2010] On page 630, in exercise 34.2, on the first line, add the words "fully continuous" after "For a".

[2/22/2010] On page 633, in the solution to exercise 34.2, on the 7th line, replace $T = 10$ with $T = 20$ and $t_1 < 10$ with $t_1 < 20$. Replace the last two lines with

$$\Pr(20 < T < t_2) = e^{-20(0.02)} - 0.076825^{0.02/0.05} = 0.31208$$

The answer is $0.04397 + 0.31208 = \boxed{\mathbf{0.35603}}$.

[3/26/2010] On page 633, in the solution to exercise 34.3, on the first displayed line, replace $(\ddot{a}_{\overline{10}|} - \ddot{a}_{\overline{t}|})$ with $(\ddot{a}_{\overline{t}|} - \ddot{a}_{\overline{10}|})$. Replace the last paragraph with:

So the loss at issue is 0 at time 32.75. This means that if the annuitant receives the payment associated with the 33rd year, the one paid at the beginning of the 33rd year, the loss at issue will be positive. The probability of that is the probability of surviving 32 years, or $1 - 32/(120-55) = \boxed{\mathbf{0.5077}}$.

[12/30/2009] On page 638, in the solution to Quiz 34-2, replace the last two lines with

$$\Pr(1.33v^{K+1} - 0.33 > -0.1) = \Pr\left(1.1^{-(K+1)} > \frac{0.23}{1.33}\right) = \Pr\left(K+1 < -\frac{\ln(0.22/1.33)}{\ln 1.1}\right) = \Pr(K+1 < 18.4120)$$

For $K < 17.4120$, death must occur before time 18. Under deMoivre, ${}_{18}q_{50} = 18/50 = \boxed{\mathbf{0.36}}$.

[12/29/2009] On page 639, equation (35.3) should have a square on the last factor:

$$\text{Var}({}_0L) = \left({}^2\bar{A}_{x:\overline{n}|} - (\bar{A}_{x:\overline{n}|})^2\right)\left(1 + \frac{P}{\delta}\right)^2$$

Also, on the line above equation (35.3), delete the word "the".

[2/1/2010] On page 640, in equation (35.4), $\bar{A}_{x:\overline{n}|}$ in the numerator should be squared:

$$\text{Var}({}_0L) = \frac{{}^2\bar{A}_{x:\overline{n}|} - (\bar{A}_{x:\overline{n}|})^2}{(1-\bar{A}_{x:\overline{n}|})^2}$$

[12/29/2009] On page 642, in Table 35.1, equation (35.3) should have a square on the last factor, as indicated in the erratum for page 639.

[2/1/2010] On page 642, in Table 35.1, in equation (35.4), $\bar{A}_{x:\overline{n}|}$ in the numerator should be squared.

[12/9/2009] On page 649, in the solution to exercise 35.10, on the first two displayed lines, put bars on each $a_{\overline{65}|}$.

[1/11/2010] On page 651, in the solution to exercise 35.20, on the last two lines, change two $P$'s to $\pi$'s.

[1/30/2010] On page 657, exercise 36.12 should start "$L(x)$ is the variance of the loss-at-issue random variable ...".

[1/10/2010] On page 665, the solution to Quiz 36-1 is incorrect. The correct solution is

Note that ${}_2p_x = 1-0.1-0.2 = 0.7$. The benefit premium is

$$\ddot{a}_{x:\overline{2}|} = 1+(1-0.1)(0.95) = 1.855$$

$$1000P_{x:\overline{2}|}^{\frac{1}{}} = \frac{1000(0.95^2)(0.7)}{1.855} = 340.5660$$

The present value of the loss at issue is

$-340.5660$ if death occurs in the first year.

$-340.5660(1+0.95) = -664.1038$ if death occurs in the second year.

$1000(0.95^2) - 664.1038 = 238.3962$ if the insured survives two years.

Since the equivalence principle is used, the variance of the loss at issue equals the second moment, which is

$$\text{Var}({}_0L) = 0.1(-340.5660)^2 + 0.2(-664.1038)^2 + 0.7(238.3962)^2 = \boxed{\textbf{139,588}}$$

[1/11/2010] On page 674, in exercise 37.11, on the last line, the 1 should be on top of 40: $1000{}_5V_{40:\overline{10}|}^{\frac{1}{}}$.

[1/11/2010] On page 681, in the solution to exercise 37.11, on the 8th and 9th displayed lines, replace 0.51675 with 0.51625 and 6.040625 with 6.046875. Three lines later, replace 6.040625 with 6.046875 and 0.012830 with 0.012817. On the second to last line, replace $P_{45:\overline{5}|}^{\frac{1}{}}$ with $P_{45:\overline{10}|}^{\frac{1}{}}$. On the last line, replace 0.012830 with 0.012817 and the final answer 5.90 with 5.94.

[1/11/2010] On page 683, the solution to exercise 37.17 is incorrect and too complicated. The correct solution is

We just need to calculate $P$ using the equivalence principle. The premium annuity is $P(2\ddot{a}_{45} - \ddot{a}_{45:\overline{20}|})$, or $\Big(2(21.875) - 13.950\Big)P = 29.8P$. Therefore

$$P = \frac{0.3}{29.8} = 0.010067$$

Using the prospective formula, the reserve is

$${}_{20}V = A_{65} - 2P\ddot{a}_{65} = 0.46 - 2(0.010067)(16.875) = \boxed{\textbf{0.1202}}$$

[1/10/2010] On page 685, in the solution to Quiz 37-2, the calculation of $A_{20}$ is incorrect. Replace the denominator $80\ln 1.05$ on that line with 80(0.05). Replace 0.251030 with 0.244956. On the next line, replace 0.251030

with 0.244956 and 15.72837 with 15.85593. On the last line, replace 15.72837 with 15.85593 and 184.29 with 185.41.

[2/12/2010] On page 688, replace the last 4 lines of the answer to Example 38A, with

$$\ddot{a}_{30:\overline{30|}} = \frac{1}{P_{30:\overline{30|}} + d} = \frac{1}{0.026 + 0.03} = 17.8571$$
$$P_{30:\overline{\frac{1}{30|}}} = \frac{{}_{30}E_{30}}{\ddot{a}_{30:\overline{30|}}} = \frac{(0.8)(0.45)}{17.8571} = 0.02016$$
$$P^{1}_{30:\overline{30|}} = 0.026 - 0.02016 = 0.00584$$

By the premium difference formula, ${}_{10}V^{1}_{30:\overline{30|}} = 15.625(0.0052 - 0.00584) = \boxed{\mathbf{-0.01}}$.

[2/3/2010] On page 689, in Table 38.2, on the line for $h$-pay whole life insurance, ${}^{h}_{k}V$ should be ${}^{h}_{k}V_x$.

[4/23/2010] On page 689, replace the three displayed lines below Table 38.2 with

$${}_{t}\bar{V}(\bar{A}_x) = \bar{A}_{x+t}\left(1 - \frac{\bar{P}(\bar{A}_x)}{\bar{P}(\bar{A}_{x+t})}\right)$$
$${}_{k}V_{x:\overline{n|}} = A_{x+k:\overline{n-k|}}\left(1 - \frac{P_{x:\overline{n|}}}{P_{x+k:\overline{n-k|}}}\right)$$
$${}^{h}_{k}V^{1}_{x:\overline{n|}} = A^{1}_{x:\overline{n-k|}}\left(1 - \frac{{}_{h}P^{1}_{x:\overline{n|}}}{{}_{h}P^{\frac{1}{}}_{x+k:\overline{n-k|}}}\right) \quad \text{for } h < k$$

[2/22/2010] On page 691, in Table 38.3, on the line for $n$-year pure endowment, change ${}_{k}V^{1}_{x:\overline{n|}}$ to ${}_{k}V_{x:\overline{\frac{1}{n|}}}$.

[2/9/2010] On page 691, in Table 38.3, on the line for $n$-year deferred insurance, $n$ premiums, in the second formula, $\ddot{s}_{x:\overline{k|}}$ should be $\ddot{s}_{x:\overline{n|}}$ so that it reads

$$\frac{{}_{n}P\left({}_{n|}A_x\right)\ddot{s}_{x:\overline{n|}}}{{}_{k-n}E_{x+n}} - {}_{k-n}k_{x+n} \qquad k > n$$

[1/10/2010] On page 695, in Quiz 38-3(i), the 1 should be over the 40: $P^{1}_{40:\overline{25|}} = 0.08$.

[1/11/2010] On page 697, in exercise 38.8, in (iv), change ${}_{30}\bar{V}_{40}$ to ${}_{30}\bar{V}(\bar{A}_{40})$. On the last line, change ${}_{20}\bar{V}_{40}$ to ${}_{20}\bar{V}(\bar{A}_{40})$ and 40th to 20th.

[3/12/2010] On page 703, in the solution to exercise 38.5, on the third line, change the double-dot on $s$ to a bar.

[2/10/2010] On page 708, in the solution to exercise 38.21, on the second to last line, change $\bar{a}_{50:\overline{10|}}$ to $\bar{a}_{50:\overline{7|}}$.

[1/10/2010] There are several errors in the quiz solutions on page 709. A replacement page is provided at the end of the errata.

[3/5/2010] On page 711, at the end of the second displayed line, change the subscript from $\angle n - k$ to $\overline{n-k|}$.

[4/26/2010] On page 713, in Example 39D(i), change 12.95 to 12.96.

[1/28/2010] On page 714, in Table 39.1, first and third Endowment Insurance lines, change ${}_{k}V_{x:\angle n}$ to ${}_{k}V_{x:\overline{n|}}$.

[2/10/2010] On page 726, in the solution to exercise 39.14, on the third displayed line, change $B$ to $\dfrac{B}{1000}$.

[1/28/2010] On page 733, formulas (40.3) and (40.5) are only valid when $t$ is an integer. Thus they would usually be written with the condition $K(x) \geq k$ instead of $T(x) > t$, although for $t$ an integer these two are equivalent.

[3/24/2010] On page 736, in exercise 40.5, while this question is the one that appeared on the old exam, the intended question was " Calculate $\dfrac{\text{Var}\left({}_tL \mid T(x)>t\right)}{\text{Var}\left({}_{t+1}L \mid T(x)>t+1\right)}$ " instead of what is on the last line.

[1/11/2010] On page 737, in exercise 40.11(ii), change $T(x)$ to $T(35)$.

[4/26/2010] On page 743, in the solution to exercise 40.21, put bars on the $A_{25}$ in the first and third displayed lines and the $A_{50}$ on the fourth displayed line.

[4/12/2010] On page 749, in exercise 41.4(iii), change ${}_{10}E_x$ to ${}_{10}E_{50}$.

[3/1/2010] On page 780, in the solution to exercise 42.5, on the first two displayed lines, remove the line in the presubscripts of $p_{xy}$.

[11/18/2009] On page 786, on the 11th line, change "to age $x+t$" to "to ages $x+t$ and $y+t$".

[11/18/2009] On page 787, in the solution to Example 43B, the values of ${}_tp_{80}$ and ${}_tp_{82}$ for $t=2$ and $t=3$ are interchanged on the 2nd, 3rd, 8th, and 10th lines of the answer.

[3/8/2010] On page 798, in the solution to exercise 43.19, on the second displayed line, change the upper bound of the integral from 2 to 3.

[3/9/2010] On page 801, on the first line of the answer to Example 44A, change "joint status" to "last survivor status".

[2/17/2010] On page 802, two lines below equation (44.3), change the $+$ to a $-$:

$$= \mathring{e}_x + \tfrac{2}{3}\mathring{e}_x^2\mu_y$$

[11/18/2009] On page 805, in the answer to Example 44E, on the first displayed line, the integrand on the left is missing a $t$ and should be $t\,{}_tp_{40:40}\,\mathrm{d}t$.

[3/18/2010] On page 816, change the last three lines of the solution to Example 45B to

$$\begin{aligned}\mathbf{E}[0.001Z] &= 2(0.24905)-0.34049 = 0.15761\\ \mathbf{E}[(0.001Z)^2] &= 2(0.09476)-0.15641 = 0.03311\\ \text{Var}(Z) &= 1000^2(0.03311-0.15761^2) = \boxed{\mathbf{8269.1}}\end{aligned}$$

[2/20/2010] On page 818, in the answer to Example 48E, 0.52652 should be 0.25634 on the two displayed lines. Replace the first displayed line with

$${}_{20}E_{45:35} = {}_{20}E_{45}\left(\frac{l_{55}}{l_{35}}\right) = 0.25634\left(\frac{8{,}640{,}861}{9{,}420{,}657}\right) = \boxed{\mathbf{0.23513}}$$

and the second displayed line with

$${}_{20}E_{45:35} = {}_{20}E_{45}\,{}_{20}E_{35}(1.06^{20}) = (0.25634)(0.28600)(1.06^{20}) = \boxed{\mathbf{0.23512}}$$

[3/8/2010] On page 828, in the solution to exercise 45.20, on the second displayed line, delete "$1-$" on the right hand side.

[11/18/2009] On page 830, in the answer to Example 46B, on the third line, change "an joint-life" to "a joint-life".

[3/22/2010] On page 832, Quiz 46-2 as stated is a difficult contingent survival question. Replace the first sentence with:

An annuity-due on (45) and (55) pays 100 per year until the later of the death of (55) and 30 years from the current date, but does not make any payments while (45) is alive.

[11/18/2009] On page 834, 3 lines before Section 46.4, change the first minus sign to an equals sign:

$$= {}_{10}E_{55}(a_{65} - a_{65:65})$$

[4/3/2010] On page 862, in exercise 36.12, 5 lines from the end, replace the first ${}^2A_{61}$, the one on the left hand side of the equation, with ${}^2A_{60}$.

[3/1/2010] On page 864, exercise 47.20, change (iii) to

The probability that an entering student fails in the first year is twice the probability that a student who completed the first year fails in the second year.

[11/18/2009] On page 875, in the answer to Example 48C, 3 lines from the end in the integrand, change ${}_{10}p_x^{(\tau)}$ to ${}_t p_x^{(\tau)}$

[3/8/2010] On page 884, in the solution to exercise 48.3, on the first and fourth displayed lines, replace $\mu_{(}^{(2)}20)$ with $\mu_{40}^{(2)}(20)$.

[3/8/2010] On page 884, in the solution to exercise 48.4, replace $\mu_{20}^{(\tau)}$ with $\mu_{40}^{(\tau)}(20)$ and $\mu^{(2)}$ with $\mu_{40}^{(2)}(20)$.

[1/5/2010] On page 910, in the solution to Quiz 49-1, in the last column of the transition probability matrix, change both 0.1's to 0.01's in both places (2nd line, 5th line), so that the matrix is

$$\begin{pmatrix} 0.9 & 0.09 & 0.01 \\ 0.3 & 0.69 & 0.01 \\ 0 & 0 & 1 \end{pmatrix}$$

Change the last sentence to

The probability of paying the fourth premium is $0.9(0.837)+0.3(0.1431)=$ **0.79623**.

[1/31/2010] On page 912, in footnote 1, change ${}_n v_t$ to ${}_t v_n$.

[4/1/2010] On page 914, on the last line of the page, change "begin" to "being".

[11/18/2009] On page 931, 9 lines from the bottom of the page, change "variables" to "variable".

[11/18/2009] On page 932, on the fourth line of the answer to Example 51B, change "number" to "numbers".

[1/11/2010] On page 934, on the first line, change "sot" to "so that".

[4/21/2010] On page 934, on the first line of Example 51H, add after "answering center": "in a nonhomogeneous Poisson process".

[11/18/2009] On page 935, the caption of Figure 51.1 should refer to Example 51H instead of 51G.

[1/11/2010] On page 935, in Quiz 51-2, change "December 1 to May 31" to "June 1 to November 30".

[4/21/2010] On page 938, in exercise 51.13, add after "amount of time" the words "in hours".

[1/11/2010] On page 943, the solution to Quiz 51-2 is incorrect. A replacement page is provided at the end of the errata.

[11/18/2009] On page 945, on the second line from the bottom of the page, add "$\mathrm{d}u$" at the end.

[4/3/2010] On page 954, on the first displayed line, add a set of parentheses around the second case: $\left(5/(20-t)\right)^2$.

[4/6/2010] On page 967, 2 lines from the bottom, change 1/2 to 1/12.

[4/3/2010] On page 969, replace the last line of the answer to Example 54F with

$$= 1 - 0.102385 - 0.213301 - 0.239964 = \boxed{\mathbf{0.444351}}$$

[4/6/2010] On page 970, in Table 54.1, 3 lines from the end, change $p_n$ to $p_k$.

[1/6/2010] On page 1033, in question 19(ii), change snnual to annual.

[4/6/2010] On page 965, change the first half of the sentence beginning 6 lines above Example 54B to

If $Y(t) = X_1(t) - X_2(t)$ with $X_1(t)$ and $X_2(t)$ independent, then the mean of $Y(t)$ is the difference of the means of $X_1(t)$ and $X_2(t)$, or $\mathbf{E}[X_1(t)] - \mathbf{E}[X_2(t)]$, but the variance of $Y(t)$ is the sum of the variances of $X_1(t)$ and $X_2(t)$, or $\text{Var}\big(X_1(t)\big) + \text{Var}\big(X_2(t)\big)$, since ...

[11/5/2009] On page 1065, in the solution to question 3, on the first line, change $\mu_{x+t}$ to $\mu_t$.

[4/26/2010] On page 1066, in the solution to question 8, replace $\overline{10-x|}$ with $\overline{40-x|}$ in all the displayed equations (twice in the first, once in the third, once apiece in the fourth and fifth) and $\overline{9-x|}$ with $\overline{39-x|}$ in the 3rd (twice), 4th (once), and 5th (once) displayed equations. The third displayed equation's left side is wrong and should be $a_{x:\overline{40-x|}} + A_{x:\overline{40-x|}}$

[11/5/2009] On page 1066, in the solution to question 10, on the first line of the table, $q_{x+t}$ should be $q_{x+t-1}$.

[11/5/2009] On page 1098, in the solution to question 10, on the sixth displayed line, add d$y$ at the end.

[11/5/2009] On page 1109, in the solution to question 19, on the sixth displayed line, change the = in the exponent to a $-$.

[12/2/2009] On page 1188, the solution to question 1 is incorrect. The correct solution is

Assume that the worker's birthday is not December 31. The worker has just turned 55 in year $x$. In year $x+4$, the worker will turn 59. We want to know the probability that the worker retires on December 31 of years $x+4$, $x+5$, or $x+6$. The probability that the worker does not retire by year $x+4$ is $l_{59}/l_{55}$, and the probability that the worker does not retire by year $x+7$ is $l_{62}/l_{55}$, so the probability that the worker retires on December 31 of years $x+4$, $x+5$, or $x+6$ is $(l_{59} - l_{62})/l_{55}$.

$$\begin{aligned} l_{59} &= 1000 - 4^3 \\ l_{62} &= 1000 - 7^3 \\ \frac{l_{62} - l_{59}}{l_{55}} &= \frac{7^3 - 4^3}{1000} = \boxed{\mathbf{0.279}} \qquad \textbf{(E)} \end{aligned}$$

The answer is far out of the range. Apparently the question writers mistakenly calculated $(l_{61} - l_{58})/l_{55} = 0.189$ and thought the answer was (C), and in fact the preliminary answer was (C).

[3/29/2010] On page 1192, the solution to question 25 should be

$S_2^1/S_2^2$ is $\sigma_2^1/\sigma_2^2$ times an $F$ variable with $(11,12)$ degrees of freedom, and $\sigma_1^2/\sigma_2^2$ is $1/k$ under $H_0$. We're given that the 95th percentile of this product is 5.2 and want the 99th percentile. The tables give 2.71 and 4.22 as the 95th and 99th percentiles of $F(11,12)$ respectively, so the 99th percentile of $F(11,12)/k$ is $5.2(4.22/2.71) = \boxed{\mathbf{8.07}}$. **(C)**

[1/24/2010] On page 1194, the columns for practice exam 6 and 7 are interchanged.

$$\text{Var}(Z_3) = 0.0158 - 0.0625 + 2(0.0820)(0.5698) = 0.046747 = v^{20}\,{}_{10}p_x\,(1 - {}_{10}p_x) \qquad (**)$$

We divide (**) by (*) to get

$$v^{10}(1 - {}_{10}p_x) = \frac{0.046747}{0.5698} = 0.082041$$

Adding this result to (*), we get

$$v^{10} = 0.082041 + 0.5698 = 0.651841$$

$$\delta = -\ln v = -\frac{\ln 0.651841}{10} = \boxed{\mathbf{0.04280}}$$

## B.15 Solutions to CAS Exam 3L, Spring 2010

The questions can be found at `www.casact.org/admissions/studytools/exam3/sp09-3.pdf`.

**1.** [**Section 21.2**] By the recursive formula for curtate life expectancy (21.4),

$$e_x = p_x(1+e_{x+1})$$

so

$$\begin{aligned} q_x &= p_x(1+q_x) \\ 1-p_x &= p_x(2-p_x) \\ p_x^2 - 3p_x + 1 &= 0 \\ p_x = \frac{3-\sqrt{3^2-4}}{2} &= \boxed{\mathbf{0.3820}} \qquad \textbf{(B)} \end{aligned}$$

**2.** [**Lesson 17**]

$$\begin{aligned} {}_{10|25}q_{30} &= {}_{10}p_{30} - {}_{35}p_{30} \\ &= \frac{s(40)}{s(30)} - \frac{s(65)}{s(30)} \\ &= \left(\frac{50}{60}\right)^{0.5} - \left(\frac{25}{60}\right)^{0.5} = \boxed{\mathbf{0.2674}} \qquad \textbf{(C)} \end{aligned}$$

**3.** [**Subsection 20.1.2**] For constant force, expected lifetime is the reciprocal of the force, so if expected lifetime is 5, then $\mu = 0.2$, and

$$1000q_0 = 1000(1-e^{-0.2}) = \boxed{\mathbf{181}} \qquad \textbf{(C)}$$

**4.** [**Lesson 22**] Starting with $l_{40} = 1000$:

$$\begin{aligned} l_{41} &= l_{40}\, p_{40} = 1000(0.8) = 800 \\ l_{42} &= l_{41} - l_{40}\,{}_1|q_{40} = 800 - 1000(0.08) = 720 \\ l_{43} &= l_{41}\,{}_2p_{41} = 800(0.63) = 504 \end{aligned}$$

With linear interpolation,

$$\begin{aligned} l_{41.75} &= 0.25(800) + 0.75(720) = 740 \\ l_{42.5} &= 0.5(720) + 0.5(504) = 612 \end{aligned}$$

and the requested probability is $(740-612)/1000 = \boxed{\mathbf{0.128}}$. **(A)**

**5.** [**Lesson 42**] The hazard rate for the joint-life status of two fans is twice the single hazard rate, or 0.196. The hazard rate for the joint-life status of three fans is $0.098(3) = 0.294$. The survival probability of at least two fans is the probability of exactly two fans (which can happen in 3 ways), or $3(p_{0:0} - p_{0:0:0})$, plus the probability of three fans, or $p_{0:0:0}$. Therefore, the answer is

$$3e^{-5(0.196)} - 2e^{-5(0.294)} = 3(0.375311) - 2(0.229925) = \boxed{\mathbf{0.6661}} \qquad \textbf{(E)}$$

## Quiz Solutions

**38-1.** From the endowment insurance, we have

$$\ddot{a}_{45:\overline{10}|} = \frac{1-0.60}{0.05} = 8$$

and from the premium difference formula, ${}_5V^{1}_{40:\overline{15}|} = 8(0.015-0.010) = 0.04$. From the paid up insurance formula, $0.04 = A^{1}_{45:\overline{10}|}(1-0.010/0.015)$, so $A^{1}_{45:\overline{10}|} = \boxed{\mathbf{0.12}}$.

**38-2.** We need to calculate $P$ using the equivalence principle. We'll need $A^{1}_{40:\overline{25}|}$ and $A_{65}$ to get the APV of the insurance. We'll need $\ddot{a}_{40:\overline{25}|}$ and $\ddot{a}_{65}$ to get the APV of the premium annuity.

$$\begin{aligned}
A_{40} &= A^{1}_{40:\overline{15}|} + {}_{15}E_{40}\,A_{55} \\
0.40 &= 0.05 + 0.7A_{55} \\
A_{55} &= \frac{0.35}{0.7} = 0.5 \\
P_{55} &= \frac{0.02(0.5)}{1-0.5} = 0.02 \\
A_{40} &= A^{1}_{40:\overline{25}|} + {}_{25|}A_{40} \\
0.40 &= 0.12 + {}_{25|}A_{40} \\
{}_{25|}A_{40} &= 0.28 \\
\ddot{a}_{65} &= \frac{1-0.56}{0.02} = 22 \\
A_{40:\overline{25}|} &= 0.12 + 0.5 = 0.62 \\
\ddot{a}_{40:\overline{25}|} &= \frac{1-0.62}{0.02} = 19
\end{aligned}$$

By the equivalence principle,

$$\begin{aligned}
2000A^{1}_{40:\overline{25}|} + 1000\,{}_{25|}A_{40} &= P\ddot{a}_{40:\overline{25}|} + 1000P_{55}\,{}_{25}E_{40}\,\ddot{a}_{65} \\
2000(0.12) + 1000(0.28) &= 19P + 1000(0.02)(0.5)(22) \\
P &= \frac{520-220}{19} = 15.7895
\end{aligned}$$

Now we'll calculate the reserve retrospectively. The only thing we're missing is $\ddot{a}_{40:\overline{15}|}$.

$$\begin{aligned}
A_{40:\overline{15}|} &= 0.05 + 0.7 = 0.75 \\
\ddot{a}_{40:\overline{15}|} &= \frac{1-0.75}{0.02} = 12.5 \\
{}_{15}V &= \frac{P\ddot{a}_{40:\overline{15}|} - 2000A^{1}_{40:\overline{15}|}}{{}_{15}E_{40}} \\
&= \frac{15.7895(12.5) - 2000(0.05)}{0.7} = \boxed{\mathbf{139.10}}
\end{aligned}$$

**38-3.** We have $P_{40:\overline{25}|}^{\frac{1}{}} = 0.48 - 0.08 = 0.40$ and can use the Three Premium Principle.

$$\begin{aligned}
{}_{25}P_{40} - P^{1}_{40:\overline{25}|} &= P_{40:\overline{25}|}^{\frac{1}{}}(A_{65}) \\
{}_{25}P_{40} - 0.08 &= 0.40(0.45) = 0.18 \\
{}_{25}P_{40} &= \boxed{\mathbf{0.26}}
\end{aligned}$$

2. Exactly one 5000 rainfall in first 5 days, no rainfall of 8000 in first 5 days, and no rainfall in last 5 days.

Rainfalls of 5000 are a Poisson process with parameter $0.8(0.2) = 0.16$ per day, or 0.8 for five days, and rainfalls of 8000 are a Poisson process with parameter 0.2 for 5 days. The probability of the first is $e^{-1} = 0.3679$. The probability of the second is $(0.8e^{-0.8})(e^{-0.2})(e^{-1}) = 0.1083$. The sum of these 2 probabilities is $0.3679 + 0.1083 =$ **0.4762**. **(D)**

# Quiz Solutions

**51-1.** There can be

1. 0 hurricanes in June and November, at least 3 in the other 4 months.
2. 1 hurricane in June and November, at least 2 in the other 4 months.
3. 2 hurricanes in June and November, at least 1 in the other 4 months.

The Poisson parameter for June and November is $2(0.5) = 1$, and the Poisson parameter for 4 months is $4(0.5) = 2$. The three probabilities respectively are

1. $e^{-1}\left(1 - e^{-2}\left(1 + 2 + \frac{2^2}{2!}\right)\right) = e^{-1} - 5e^{-3}$
2. $e^{-1}\left(1 - e^{-2}(1+2)\right) = e^{-1} - 3e^{-3}$
3. $e^{-1}\frac{1^2}{2!}\left(1 - e^{-2}\right) = 0.5e^{-1} - 0.5e^{-3}$

The sum is $2.5e^{-1} - 8.5e^{-3} =$ **0.4965**. If you calculated the three probabilities separately, you should've gotten 0.1189, 0.2185, and 0.1590 respectively.

**51-2.** The mean number of tropical storms for a season is

$$m(0.5) = \int_0^{0.5} (24t - 48t^2)\mathrm{d}t = 12t^2 - \frac{48t^3}{3}\bigg|_0^{0.5} = 1$$

We would like $n$ such that $\sum_{i=0}^{n} p_n > 0.9$, or $e^1 \sum_{i=0}^{n} p_n > 0.9e = 2.4465$. Summing up $ep_n$:

$$1 + 1 + 0.5 = 2.5$$

so $n =$ **2** is the smallest number for which the probability of $n$ or fewer hurricanes is greater than 90%.